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The Right Hon. LORD MONCREIFF, President, in the Chair.

The following Communications were read:—

1. The Trigonometrical Survey of Palestine. By Lieut. Claude Reignier Conder, R.E.

The survey which forms the subject of the present paper extends over an area of 6000 square miles, bounded by the Jordan on the east, the Mediterranean on the west, the river Leontes and the springs of Jordan near Dan on the north, and the desert of Beersheba on the south. Within these limits a complete triangulation, with two bases each about four miles long, has been extended, and the whole of the country mapped to the scale of one inch to the mile. The work occupied five years in the field, and nearly three years more in preparing the results for publication.

The trigonometrical survey was first commenced in October 1871, when a party consisting of only four Europeans took the field. This expedition was sent out by the Palestine Exploration Fund, under the auspices of which society, Captain Warren, R.E., had previously been employed in his adventurous explorations at Jerusalem during the years 1867 to 1870.

The officer selected for the command of the survey party was Captain Stewart, R.E., of the Ordnance Survey. Unfortunately the

expedition reached Palestine in the most unhealthy season of the year, and their leader was invalided home in January 1872. The two surveyors, Sergeant Black, R.E., and Corporal Armstrong, R.E., were thus left under the care of Mr. C. F. Tyrwhitt Drake, who had been attached to the expedition as linguist and archæologist.

On the return of Captain Stewart, the Committee of the Palestine Exploration Fund honoured me with the offer of the command of the survey party, and I reached Jaffa on the 8th of July 1872. In the meantime the survey had been successfully started, and after measuring the first base in the plain of Sharon near Ramleh, the triangulation was extended first to Jerusalem and then northwards to Nablus or Shechem, the detail being at the same time filled in over an area of 500 square miles.

The success of this part of the work was due not only to the zeal and skill of the two non-commissioned officers, but also in a great measure to the tact and experience of Mr Tyrwhitt Drake, whose knowledge of Syrian manners and language was invaluable.

The method of conducting the survey was soon developed into a routine which was preserved throughout the course of the field operations. The camp having been fixed in a convenient position and as centrally as possible, with reference to the proposed work, the triangulation was first extended. The highest points within a radius of ten or twelve miles from camp, were visited, and at the points selected cairns of stone seven to ten feet high were built up and carefully whitewashed. In some cases the domes of the sacred tombs formed valuable stations, and in the more wooded parts of the country it was necessary to clear away the brushwood, leaving a lofty stack of branches bound to a central pole marking the instrumental station.

The triangles thus constructed varied from five to fifteen miles side according to the character of the country. Several very long lines were also observed, and from the ends of the bases astronomical observations were taken to fix the meridian lines. The observations as calculated at Southampton in 1877 showed an error rarely exceeding thirty feet in the mile, which it is unnecessary to remark is not visible on paper to the one-inch scale.

From the trigonometrical stations observations were also taken to all prominent objects within the field. It was found, however,

that the effects of mirage rendered these observations untrustworthy at a distance of more than about six miles from the station. The sacred tombs, solitary trees, village towers, and other conspicuous landmarks were fixed by the intersection of the various angular directions thus observed, and these served as secondary points in the work which next followed.

The trigonometrical observations were calculated in camp and laid down on rough sheets. Tracings were then prepared for each of the surveyors, and the district surrounding the camp subdivided. Each surveyor, accompanied by a local guide, then proceeded to fill in the detail of the sheets with the aid of the prismatic compass and the names of the various features were carefully collected from the guides, and verified as far as possible by reference to independent witnesses.

The detail which was shown may be seen on the lithographed sheets. It includes towns and villages, ruined sites and isolated buildings, springs, wells, cisterns and aqueducts, enclosures and roads, all the principal isolated trees, and the cultivation or natural growth of the country, rock-cut tombs, vineyard towers, wine presses, and other traces of ancient cultivation, as well as the dry torrent beds and perennial streams which form the natural, boundaries of the ancient divisions of Palestine.

The collection of the names was one of the most delicate and important parts of the work. It is well known that the nomenclature of Palestine, so far at least as the sites of towns and villages are concerned, has remained almost unchanged from a very remote period.

The reasons for this preservation of the Hebrew nomenclatnre will be mentioned later; but it may be noted that one of the original objects for which the survey was undertaken, was that of collecting ancient names hitherto unknown, for the purpose of assisting in the identification of biblical sites, especially in those districts of the country previously almost unexplored.

In order to secure the correct orthography of the nomenclature a native scribe was attached to the party, and in order to secure the correct application of the names it was made a rule only to inquire on the very spot. The site being thus ascertained, the correct pronunciation was obtained on the evening of the same day from the local guide on the return of the surveyor, and was written down by the scribe. 10,000 names were thus collected in 6000 square

miles, and although a large proportion of these are of little value, the result of this systematic examination of the nomenclature has been the recovery of about 150 ancient sites, which are newly identified with places mentioned in the Bible, while a yet larger number of names connected with the Byzantine and Crusading history of Palestine have also been recovered.

The elevations of the trigonometrical stations above the sea were obtained by vertical angles with the theodolite, and checked by means of measured heights along the coast. The error in these heights does not appear to exceed three or four feet on an average.

Other elevations were observed with aneroid barometers, and checked by readings taken at bench marks or at trigonometrical stations and by readings of the mercurial barometer in camp. A line of levels had been run from Jaffa to the Dead Sea in 1864 by Captain Wilson, R.E., and with this our trigonometrical observations agreed very well. The summer level of the Dead Sea was thus fixed at 1292·5 feet below the Mediterranean, the surface in summer being about fifteen feet lower than in winter.

In 1875 a second line was run by the survey party from the Bay of Acre to the Sea of Galilee, a distance of about thirty miles, and the level of this lake was thus fixed at 682·5 feet below the Mediterranean. This piece of work was carried out under a special grant from the British Association, former estimates of the depression having ranged from 300 to 600 feet.

By means of these levels and of observations to trigonometrical stations between the two lakes and farther north, the fall of the Jordan was determined throughout the whole of its course.

The rate at which the survey was carried on increased gradually as the party became more accustomed to the country and better acquainted with the language and customs of the natives. At first it did not exceed fifty square miles in a month, but after leaving Nablus an average of 100 was obtained. In 1873 this was increased to 150, and in 1874 to upwards of 200. The party was strengthened in the latter year, when an average of 270 square miles per month was attained, and kept up almost to the end of the survey. The most rapid piece of work was the survey of the Desert of Judah. The triangulation being very large and the detail less close than in the cultivated districts, the survey of this desert was completed in ten

days, the area being 330 square miles of very difficult mountain country.

The representation of the hill features was of necessity less exact than might be possible on the one-inch scale, but so far as the ridges and prominent features are concerned it is supposed to be accurate. The general slopes of the mountain sides were determined with a pocket level, and separate sheets of hill shading were prepared and carefully inked in before leaving the camp for a new district.

The preceding notes will, it is hoped, give some idea of the character of the survey work. The rapid rate of progress had many advantages, amongst which was the economical character of the work, the map costing in the field not more than one penny per acre of ground surveyed.

In addition to the map work many other kinds of information were supplied. The ruins were explored, plans of all interesting places, special surveys of towns and ruined cities, and notes on the archæology of the land, were regularly collected. The geological structure of the country was observed, collections of fossils and of lithological specimens were made, birds were shot and stuffed, notes on the traditions, manners, and language of the peasantry were recorded. Photographs of interesting places were also made, and a regular series of meteorological observations was kept for three years.

It was also necessary to study carefully the literature of the subject, in order that intelligent explorations might be attempted, and special observations taken in connection with the numerous controverted sites of importance throughout the country. The winter seasons were employed in reducing to order the field observations, and in preparing, by study of the literature, for the examination of questions connected with the topography of the district next to be surveyed.

The results of these explorations are not as yet fully published. The lithographed sheets now on the table have been prepared at the Ordnance Survey Office, Southampton, from MSS. worked out in England after the return of the survey party. An engraved map to a smaller scale ($\frac{3}{8}$ inch per mile) is also in course of preparation, and the proofs seem to promise that the finished work will be of excellent execution.

A memoir has also been prepared by the survey officers to accompany each of the 26 sheets of the one-inch map, and this great collection of notes is now going through the press under the editorship of Col. Wilson, R.E., whose absence on special duty in Anatolia has, however, unfortunately delayed the publication.

The memoir is divided into four sections. The first descriptive of the country, its natural features, its towns and villages, with special notes on the ancient history of the various sites. The second section deals with the archæology of the sheet, every ruin being described, with accompanying plans and sketches. The third section consists of translated name indexes, giving the Arabic lettering and the connection of ancient with modern names. The fourth section includes notes on the ethnology of the sheets. Special papers on the physical geography, architecture, nomenclature, and archæology of the various districts, on the geology of the country and on the ethnological peculiarities of the natives, will, it is hoped, be added, and the whole memoir will probably exceed 1000 pages quarto of print, with more than 100 special plans and surveys.

Some account may now be desirable of the history of the undertaking, which was not free from vicissitudes, and of the most interesting results of the exploration apart from the survey map itself.

Leaving the well-watered vale of Shechem on the 16th August 1872, the survey party proceeded northwards, and in September encamped at Jenin on the south edge of the great plain of Jezreel or Esdraelon—a plateau divided from the Jordan valley by the ridge of Gilboa, and from the plain of Sharon by the spur which runs north-west to the Carmel promontory. In this plain the second or check base was measured, and a new triangulation extended from it in a very satisfactory manner. The hills round Nazareth were surveyed during the autumn months, and the party wintered safely in the German colony at Haifa under the slope of Carmel.

Several instances of molestation occurred during this campaign to different members of the party, the most serious being an assault on Sergeant Black by the young men of a village, who were firing at a mark, but sent several bullets in an opposite direction, which fell at the sergeant's feet as he was taking angles. These offenders were, however, punished by fine and imprisonment.

In the early spring of 1873 the expedition turned south, and commenced to fill in the country lying between the sea-shore and the mountains forming the back bone of Palestine, which had been previously surveyed. The plain of Sharon was thus visited during the most healthy season, and a pleasant time was spent in a district previously but little known.

Among the most interesting places visited was Athlit, the ancient Castel Pelegrino—a fortress of the Templars, held by the Christians to within a few months of the fall of Acre in 1291, and one of the best preserved examples of crusading architecture in Palestine. On Carmel the remains of an unknown synagogue were explored; at Cæsarea the ancient temple built by Herod the Great in honour of Augustus was discovered though not completely examined. The identification of Antipatris with the ruins of Ras el 'Ain was confirmed by the survey operations; and the great wood called Drumos by Strabo was for the first time thoroughly explored, at the north end of the Sharon plain and at the foot of Carmel.

The party rested during the summer months on the heights of Anti-Lebanon above Damascus, and in October the ascent of Mount Hermon was accomplished and a night passed on the summit; the latitude, longitude, and elevation of the highest peak, 9200 feet above the Mediterranean, being carefully determined by trigonometrical and astronomical observations.

Leaving the Anti-Lebanon on 24th of September 1873 the party marched to Beirut, and thence down the whole coast as far as Jaffa—the distance of 220 miles being accomplished in seven days. The survey was next extended southwards from the former limits —the Judean hills round Bethlehem being carefully examined This part of the work was of great interest in consequence of the number of biblical sites included in the district. The most valuable discoveries were perhaps those of the rock Etam, in a cleft of which Samson hid from the Philistines, and of the probable site of the village Emmaus, sixty stadia from Jerusalem, at the present ruin of Khamasa, south west of the Holy City.

East of Bethlehem the desert of Judah was next entered, and the survey extended to the cliffs west of the Dead Sea. In this desert the name *Suk*, applying to a mountain where the scape goat used to be destroyed in later Jewish times, was found still surviving

at the distance from Jerusalem given in the Talmudic writings. The fatigue of the work was here much increased by the great power of the sun, in a district entirely bare of trees and composed of steep ridges of white marl with precipitous limestone gorges, the only water obtainable being warm and brackish and at times very scanty in quantity.

On the 15th of November the broad plains of Jericho were reached, and a district of about 200 square miles north of the Dead Sea was surveyed. No traces of the Cities of the Plain were, however, found, and the conclusion resulting from a careful examination of the ground was that these towns probably stood east of the Dead Sea or higher up the Jordan valley, where fresh water would be found to supply them. The plains of Jericho are deficient in water supply, and the soil is so deeply impregnated with salt that it seems impossible that it should ever have been cultivated within historic times.

At Jericho the expedition suffered severely from an epidemic of malarious fever, which so weakened the party as to render field work impossible during the winter. Mr Tyrwhitt Drake narrowly escaped with his life, and only three members of the party of twenty-five individuals remained unaffected. The winter months were passed at Jerusalem, and the weather experienced was unusually severe, seven falls of snow occurring in the hills, while the Jordan valley was flooded and rendered impassable.

In the end of February 1874 the expedition again took the field, and the most difficult and dangerous part of the survey was, during the next two months, carried through successfully.

This task consisted in the exploration of the Jordan valley between the plains of Jericho and the Sea of Galilee—a district entirely uncultivated and inhabited only by nomadic Arabs living in tents. The supplies were brought down from Jerusalem or Shechem, a distance often of two days' journey, and the party was obliged to trust entirely to itself for defence, as the Turkish government exercises but little control over the Bedawin. The work was interrupted by constant storms, and the oppression of the atmosphere at a level 1000 feet below the sea was found extremely trying. The water supply was very uncertain, and for ten days the party were obliged to rely on the salt springs of Wady Maleh. During

the last two weeks the power of the sun became so great that work was only possible in the early morning, for the party remained in valley even after the Arabs had retired to the hills. The exhaustion due to this campaign necessitated a long rest for the whole party, and the expedition experienced a sad and serious loss in the death of Mr Tyrwhitt Drake, who sank under a second attack of fever brought on by the malaria of the valley and the toils and privations of the survey work.

It was not until the 5th October that the expedition was again able to take the field. Lieutenant Kitchener, R.E., was appointed to take the place of Mr Drake as second officer for the party. At this time more than half the survey (3500 square miles) had been completed, including all the country between Bethlehem and Nazareth, and it was determined to complete the southern portion of the work, 1200 square miles, before attempting the survey of Galilee: the autumn of the year was consequently passed in the hills of Hebron and the desert of Beersheba. The season was unusually sickly, and the mortality in the plains was in some villages not less than 50 per cent. of the native peasant population. In the high mountains the party were, however, comparatively safe, although it became necessary to invalid one of the most valuable members (Sergeant Black) during the winter. An attempt was made to push the work through the desert west of the Dead Sea about Christmas time, but the expedition was driven to shelter by a succession of violent gales which nearly wrecked the camp.

In the spring of 1875 a very light expedition was organised for the exploration of the desert. The invalid members of the party, including Lieutenant Kitchener, remained in Jerusalem with the baggage, and accompanied by two corporals I set out carrying only the barest necessaries, with food and provender for two days at a time. The work was pushed on with the greatest possible rapidity, and as before stated 330 square miles were mapped in ten days. The shores of the Dead Sea, here girt with cliffs 4000 feet high, were visited, and a special survey made of the famous fortress of Masada. The Arab tribes were found in a very disturbed condition, in consequence of recent tribal conflicts, and this portion of the work was perhaps the most adventurous episode of the survey.

In the middle of March the survey of the low hills and plains of

Philistia was commenced and carried successfully to the southern limit of the map at Gaza. The site of Adullam and of the famous cave of the same name was determined for the first time, the ruins of Ascalon were examined, and the sites of Ekron, Gath, Ashdod, and Jamnia surveyed. The work was comparatively easy in this open and cultivated district, and the total area surveyed was thus quickly raised to 4500 square miles.

After a month's rest at Jerusalem the party marched northwards in July, and the survey of Lower Galilee was commenced, including the line of levels from the Mediterranean, which necessitated an encampment close to an unhealthy swamp north of Nazareth.

The expedition then moved northwards to Safed, the intention being to carry on the work until the winter in the mountains of Upper Galilee, leaving only the upper Jordan valley and the plain of Phœnicia to be completed in the spring of 1876.

Unfortunately the work was for a time completely stopped by a combination of difficulties. The party was attacked on the 10th of July by a mob of fanatics at Safed, and for a short time was in considerable danger; the prompt assistance sent by the Turkish Governor rescued us just as resistance began to become no longer possible, but scarcely a member of the expedition escaped without more or less serious injury.

It was impossible after this to carry on the work until the assailants had been punished, and the party consequently retreated to Carmel. The principal offenders were tried and imprisoned, and a fine of £270 was inflicted on the town. At this time, however, the whole expedition succumbed to fever, partly due to the injuries received, and a serious outbreak of cholera throughout Syria rendered it prudent to withdraw the party from the country.

The members of the expedition continued to suffer from fever on their return to England, and the field work was consequently suspended during 1876. In 1877, as my own health continued to be unsatisfactory, it was considered best to divide the party. Half of the expedition was sent out under Lieutenant Kitchener, to complete the 1300 square miles which remained to be surveyed; the other half was retained under my direction, to work out in London the results already obtained, representing four fifths of the whole work.

The survey of Upper Galilee was successfully and peacefully carried out by Lieutenant Kitchener, and as the population of the district was chiefly composed of Christians and Jews no further serious difficulties were encountered. The most valuable discoveries in this part of Palestine were the various cromlechs found by the survey party,—the first undoubted specimens of rude stone monu ments as yet discovered west of Jordan.

The construction of a large scale map was not the only duty of the survey party. Information was also expected on all antiquarian and scientific questions which it might be possible to examine. Among these the principal results connected with the ethnology, geology, zoology, physical topography, and architecture may be briefly noticed, and a few words added in conclusion respecting some of the biblical sites discovered by the survey party.

The great explorer, Dr Robinson, was one of the first writers who called attention to the conservation of ancient names and traditions among the Syrian peasantry. The collection of 10,000 local names during the course of the survey, not only resulted in the addition of many new sites to those already known, but served to throw light on the reason of the preservation of ancient Hebrew names almost unchanged in the modern nomenclature. The language of the peasantry proves to be much nearer to Aramaic or even to Hebrew than to the pure Arabic of Arabia proper. Not only does the pronunciation of various letters and words reproduce the Aramaic sounds, but many words in common use among the peasantry are of pure Aramaic origin, and are not used, or even in some cases not understood, by the townspeople who employ the more modern Arabic equivalents.

As a single instance the word *Jurn* may be noted. In Arabic it means a trough, but among the peasantry it signifies a threshing-floor, like the Hebrew *Goran*. The Arabic word used by the towns-folk and educated classes to signify a threshing-floor is *Nadir*, and it was not until after some time had elapsed that we discovered the meaning attached by the peasantry to the word *Jurn*. Many other instances might be quoted; but the general result seems to be that the peasant language in Palestine is almost unchanged since the times of Jewish domination. The preservation of the ancient nomenclature is thus easily explained, and the explanation is

confirmed by the fact, that the ancient names are as a rule irretrievably lost in districts inhabited by the Bedawin who immigrated at a late period from Arabia to the Syrian deserts.

Not only the language but the customs, dress, and religion of the peasantry are extremely archaic. The old worship of high places is still preserved among the villagers, sacrifices not in accordance with the doctrines of Islam are offered to local divinities, lamps are lighted, votive offerings suspended, and solemn dances and processions celebrated at the innumerable shrines which are found on every high mountain and under every large tree.

So close is the correspondence between the habits of the peasantry and the description of the habits of the indigenous population in Jewish times, that the Fellahin of Palestine may apparently be without improbability considered the descendants and representatives of the ancient Canaanite tribes.

The Zoology of Palestine has been made a special study by the well-known explorer Canon Tristram, and we had no hope of being able to add in any material degree to his discoveries. Fortunately, however, we were able to determine the existence of a species of deer not previously known to inhabit Palestine. By the natives it is called *Yahmûr*, a word identical with the Hebrew term rendered "fallow deer" in the Bible. A specimen of the *Yahmûr* was brought to us by the Arabs of Carmel, and the skin and bones were sent by Mr Tyrwhitt Drake to the Museum at Cambridge. The animal was there pronounced indistinguishable from our European roebuck, and we were thus able to ascertain the actual species of game which furnished the tables of Solomon with savoury venison.

The Geology of Palestine presents features of unique interest connected with the extraordinary depression of the Jordan valley. The country has never yet been thoroughly examined by a professional geologist, and although much has been done by M. L'Artet and by Canon Tristram, much still remains requiring skilled judgment to explain. The attention of the survey party in respect of this question was principally devoted to the general structure of the country and the distribution of the main divisions of the strata.

The hill country of Palestine is formed by a steep anticlinal, the strata belonging principally to the period of the Lower Chalk, and

including soft white marls and limestone overlying a hard crystalline dolomitic limestone of the Neocomian series.

Batches of nummulitic limestone belonging to the early Tertiary beds are found in Galilee, and on Ebal, Gerizim, and Olivet. On the western slopes of Lebanon and on the east side of the Jordan valley the Nubian sandstone belonging to the time of the Greensand is found, but this formation never appears west of Jordan.

The dip of the strata along the Jordan valley was very carefully noted during the prosecution of the survey. In every case a very sudden contortion of the strata was observable, the dip being eastwards or south east, and in places faults were found extending north and south. It was clear that the original depression had taken place after the Chalk period, and the basaltic outbreaks which surround the Sea of Galilee, and cover 500 square miles east of the lake, also belong apparently to the time of the first breakdown of the chasm in the early Tertiary period.

The appearance of the sandstone east of the valley bottom is considered by L'Artet a conclusive proof of the fact, that the whole depression is due to a fault running north and south for 150 miles, and giving a fall of 3500 feet from the springs of Jordan to the bottom of the Dead Sea.

The remains of an ancient beach were discovered by the survey party north of the plains of Jericho, and again south of the Sea of Galilee. Near the Dead Sea other beaches are visible at different levels, and it is clearly evident that the present valley was once occupied by a chain of great salt lakes, the surface of which was about on the same level with the Mediterranean, and which have at a comparatively recent geological period undergone a process of desiccation until they are now only represented by the smaller sheets of water known as the lakes of Merom and Tiberias and the Dead Sea, the extreme saltness of the latter being due to the gradual washing down of the chlorides from the basins now dry, a process which seems to promise the final consolidation of the Sea at a remote period into a bed of 500 square miles of salt.

The questions connected with the climate and physical geography of Palestine, its ancient fertility, its present desolation, and the possibility of its future restoration, are of still higher interest, and the survey seems to have thrown considerable light on these subjects also.

The impression produced on first entering Palestine is that of a barren country and of desolate ruins representing a former condition of prosperity. It must not, however, be supposed that the soil is wanting in fertility. The luxuriant growth of weeds and wild bushes sufficiently attests the richness of the land. In those districts where the soil consists of porous chalk, and where the water sinks down to the underlying impervious strata, the bareness of the country is very remarkable. In the higher mountains, where the dolomitic limestone is denuded, the western ridges are thickly clothed with copses of mastic and dwarf oak. In Sharon and Lower Galilee extensive woods of oak still exist; and although a great destruction of forest (which existed even as late as the twelfth century) has apparently occurred in some districts, there is a corresponding spread of the thickets in other parts of the country, where the sites of ancient vineyards and orchards are found overgrown with thick copses.

There does not appear to be any good foundation for the popular theory of a great diminution in the rainfall of the country. The average fall is now about twenty inches in the year, and all the famous springs noticed in the Bible are found still to yield a good supply. There are twelve considerable perennial streams in Palestine besides the Jordan, and many districts, such as the Hebron hills and the lowlands of Galilee, are plentifully supplied with springs. In the chalk districts no change in the supply can apparently have taken place within historic times, and the great number of cisterns and tanks, many of which are certainly of immense antiquity, gives evidence that it was necessary, even in the earliest historic period, to provide a large amount of storage for rain water in the districts not naturally supplied.

The change which has actually occurred in the climate and condition of the country seems to be less important than is sometimes supposed, and appears to be due principally to depopulation and to the decay of the ancient cultivation. The malaria of the lowlands is plainly traceable to the absence of proper drainage, and to the destruction of the ancient works of irrigation, many of which date back at least to the Christian era.

The swamps formed in the plain of Sharon are due to the filling in of ancient rock-cut channels, which once conducted the drainage

of the mountains to the sea, and the miasma in many cases arises from the stagnation of water in the torrent beds, which might with very little difficulty be drained into the sea.

Throughout Palestine, also, there are evidences of an extensive and careful cultivation now entirely abandoned, and of a population which has been estimated at not less than ten times the number of the present inhabitants. The sides of the hills are carefully terraced, though now often only growing thorns and thistles. Ancient wine presses and rude stone orchard towers are encountered in every direction, often on the sides of hills now entirely uncultivated. The ancient ruined towns and villages, so thickly strewn over the country, number more than ten times the present total of inhabited villages. The population, which does not exceed three millions for all Syria, is entirely inadequate for the cultivation of the country, and the villages are thus found standing in tracts of plough land or orchards surrounded on every side with waste ground or thick copse.

The riches of Palestine appear now as of old to be principally agricultural. The quality of the corn, wine, and oil is not inferior to that of even the south of Italy, and it can scarcely be doubted that, should any circumstances lead to the development of the natural wealth of the country, Syria might become an important source for the supply of the three products above mentioned.

The restoration of the country to a condition of prosperity depends, in short, not on any change in its climate, rainfall, or vegetation, but on the establishment of a just government, the liberation of the native peasantry from unjust taxation, violence, and oppression, and on the establishment of a condition of security which might induce the Jewish and other local capitalists to invest their money in the cultivation and irrigation of the land, in the development of its trade, and in public works which are at present entirely non-existent.

The examination of the ruined sites throughout the country formed one of the most important occupations of the survey officers. A note was made of every ruin which could be found, and a sketch or plan of every object of interest. The hopes which were naturally entertained of the discovery of remains belonging to the Jewish or Phœnician period were, however, doomed to disappointment, and the conclusion to which it seems necessary to submit is that the Jews

were not a people of great architectural genius. Large numbers of Byzantine and Crusading buildings were examined, and many structures previously attributed to an earlier period were clearly proved during the course of the survey to belong to the late times of Christian domination in the country.

With exception, indeed, of the rock-cut tombs and of the great walls of the Jerusalem Haram, not a single building was found which could be attributed to an origin earlier than the times of Herod the Great. The walls of Masada, the aqueducts of Cæsarea, the colonnade at Samaria, and the great building at Herodium, appear to be the work of this monarch; but the idea which finds expression in many books on Palestine, that all masonry with a sunk channel or draft round the stones is of Jewish or Phœnician origin, was plainly disproved by the observations taken during the course of the survey.

The ancient sepulchres formed a study of the greatest interest and importance, as serving to indicate roughly the date of ruins where they occur. The earliest Jewish tombs with *Kokim* or narrow graves running in from the walls of the chamber, were found to have been superseded about the time of Christ by another form of sepulchre, in which a rock-cut sarcophagus was excavated at the side of the chamber, while a cylindrical stone took the place of the older hinged or sliding door. The rock-cut tombs of the Christian period, fitted for the burial of two bodies—man and wife—are again quite distinct in form, consisting of graves sunk in the flat rock and covered with a great stone. Dated inscriptions were found on many of these tombs, with Christian emblems and leaden coffins.

Some light has also perhaps been thrown on the vexed question of the length of the Jewish measure called *Ameh* or cubit, by the careful measurements of the synagogues and of the Temple buildings at Jerusalem.

According to the Talmudists the cubit measured the length of 48 barley corns, which by measurement of barley corns in Palestine would represent 16 English inches very closely.

It was found that the pillars of the synagogues had in many cases a total height of 160 inches, or 10 cubits of 16 inches, the bases being 16 inches (1 cubit), the capitals 8 inches (half a cubit). In 1873 I was so fortunate as to discover a part of

the Haram Wall at Jerusalem not previously examined, where buttresses of ancient masonry are built at intervals. The interval from centre to centre was 160 inches or 10 cubits, and the dimentions of many of the great stones are in the same way multiples of a unit of 16 inches, which it is thus natural to conclude represents very closely the length of the medium Jewish cubit.

The limits of the present paper will not allow of any account of the exploration of Jerusalem. The survey of that city had been previously executed in 1864, by Col. Wilson, R.E., and the excavations of Captain Warren, R.E., had placed the topographical controversies on an entirely new footing. A certain amount of additional information was collected by the survey party, including 150 observations of the rock-levels in the city, which have an important bearing on some of the disputed questions, but the subject is too large to be further noticed in this paper.

As has been already stated, the identification of ancient sites, especially those connected with biblical history, formed one of the principal objects contemplated in undertaking the Palestine survey. The results in this field of research have been perhaps more satisfactory than could have been expected. About 170 new identifications have resulted from the survey, and it is satisfactory to be able to say that most of these have been well received by students of the subject, and pronounced valuable by good authorities. The number represents about two-fifths of the total of biblical sites now identified, the remainder being the results of the labours of the famous travellers Burckhardt, Robinson, Vandevelde, and others.

Among the places thus newly recovered, or concerning which fresh information has been collected by the survey party, may be mentioned the royal Canaanite cities of Hazor and Debir, Adullam, Lachish, and Megiddo, with the New Testament towns Emmaus and Salem. New information has also been collected as to Capernaum, which the survey officers are inclined to place at the site proposed by Dr Robinson, called *Minieh*, rather than at the traditional site of *Tell Hûm*. It would, however, be impossible to enter at length into the various interesting questions connected with these sites.

A single example of a survey identification may, however, be noticed in conclusion of this paper, as being perhaps the most interesting result of the survey of Palestine, namely, the recovery of

the supposed site of the Bethabara of the New Testament, a place where John the Baptist met our Lord, and where it is supposed that the baptism of Christ occurred.

Since the fourth century A.D., the traditional site of Bethabara has been always shown at the most southern ford of Jordan, due east of the present Jericho. This spot is annually visited by crowds of devout Russian or Syrian Christians, and the scene of their immersion in the river is extremely picturesque and has often been described by travellers. There is not, however, any conclusive evidence that the site so pointed out is genuine; the name Bethabara does not survive in the vicinity, nor is the place mentioned by any writers before the fourth century.

There is, moreover, a fatal difficulty in identifying the true and the traditional site; for it is clear from the Scripture narrative that Bethabara lay at not more than a day's journey from Cana of Galilee, which was situated north of Nazareth. We are thus limited to a distance of about twenty miles from Cana (which is probably the present village called *Kefr Kenna*), whereas the traditional site is no less than eighty miles from Cana, a distance representing three days' journey rather than one.

The name Bethabara is a compound of *Beth* "a house" and *Abara* "a passage" or ford. The village stood beyond Jordan, and apparently took its name from a neighbouring ford of the river.

The fords of Jordan were carefully examined during the course of the survey in the Jordan valley. No less than forty fords were found between the Sea of Galilee and the Dead Sea, of which only six are marked on former maps. Their names, which are mainly descriptive, were obtained, and care taken to ascertain as far as possible the exact positions. Some of the fords are only passable in summer or autumn, but others, to which the main roads lead, are practicable at all times except during heavy floods.

Among the fords, one and one only was found having the name 'Abârah, identical in form and in meaning with the name of the ford whence Bethabara took its title. This passage of the river, discovered by Sergeant Black in the ordinary course of the survey work, is situated about twelve miles south of the Sea of Galilee, at the place where one of the main roads from Lower Galilee crosses over into the district of Bathania or Bashan.

The name thus recovered is not a mere descriptive title in common use among the Arabs. No other ford is so called as far as could be ascertained by careful questioning, and the word 'Abârah does not occur again among the 10,000 names collected within the limits of the survey.

The distance from the ford 'Abârah to the probable site of Cana of Galilee is about twenty-two English miles, the road being the shortest and easiest leading from that town to any part of Jordan. There is thus a possibility of journeying in a single day between the two places, which, as before mentioned, agrees with the account given of Bethabara in the gospel narrative, but which is not in accordance with the position of the traditional site near Jericho.

This discovery is a fair sample of the biblical results due to the survey. The discoveries of similar character connected with the Byzantine and Crusading history of the country are not less numerous or interesting; but I hope that what has now been said may serve to show the aims of the work, and that—when the difficulties connected with its execution are borne in mind—the results may be considered adequate for the time and money expended on the survey.

In conclusion, I would venture to say that the success of the work should have a peculiar interest for Scotsmen, for although the leaders could not claim Scotch descent, it is to the zeal and faithfulness of the two sergeants, Black and Armstrong, that the thoroughness and accuracy of the survey are in great measure due, and both these non-commissioned officers, as well as others of the staff, were natives of the north side of the border.

2. On Minding's System of Forces. By Professor Chrystal.

(*Abstract.*)

Minding has proved a remarkable theorem concerning a variable system of forces defined as follows, the points of application of the different forces, and their magnitudes are given, while the directions are such that a pencil of rays through any given point parallel to them moves as a rigid body.

Besides Minding's original investigation, several others have been

given since. The last of these, due to Professor Tait, rests on purely quaternion methods, and is so elegant and concise that I was led to reinvestigate the whole subject by ordinary methods in the hope that the analysis might have some points of interest. Two methods of arriving at Minding's result are given, and a variety of other conclusions are arrived at by means of the second method sufficient to indicate the course of a full investigation of the complex formed by the central axes, and of the congruency formed by the single resultants of Minding's system.

First Method.

The components of force and couple are found in terms of the Rodrigues co-ordinates $\lambda\mu\nu$, which determines the position of the rigid pencil representing the direction of the forces.

The equations to the single resultant are then found in terms of two constants g and h, and the parameters $\lambda\mu\nu$.

Equations are then deduced for the values of $\lambda\mu\nu$ corresponding to a ray passing through a point xyz. Eliminating μ and ν a biquadratic is found for λ. The system of resultant rays therefore forms a congruency of the fourth order.

This biquadratic becomes wholly indeterminate for points on the real focal conics of the ellipsoid

$$\frac{x^2}{g^2+h^2} + \frac{y^2}{h^2} + \frac{z^2}{g^2} = 1 . \quad . \quad . \quad \text{(A)}$$

Some farther discussion leads to the conclusion that the resultant rays of Minding's system is identical with the congruency of rays that intersect the two focal conics of (A).

Second Method.

If $\xi\eta\zeta$ be the co-ordinates of the feet of the perpendicular from the origin on any ray whose direction is (λ, μ, ν), and ρ the length of that perpendicular, it is shown that

$$\rho^2 = g^2\mu_1{}^2 + h^2\nu_1{}^2 \quad . \quad . \quad . \quad \text{(B)}$$

$$\rho^4 + g^2\eta^2 + h^2\zeta^2 = g^4\mu_1{}^2 \div h^4\nu_1{}^2 \quad . \quad . \quad \text{(C)}$$

(B) is true for central axes generally, and determines a complex of the second order which they form. Both (B) and (C) are true for

the rays of single resultant, and are the twofold relation which determine a congruency with which they are identical.

A discussion is given of the complex determined by the relation

$$\rho^2 = f^2\lambda_1{}^2 + g^2\mu_1{}^2 + h^2\nu_1{}^2 \quad . \quad . \qquad (D)$$

of which (B) is a particular case.

The equations to Plücker's complex cone and equatorial and meridian surfaces are given, and various loci connected with the complex are discussed.

A method of exploring the complex by means of central radii is then given.

It is found that the stretch on any radius that is intersected by rays of the complex perpendicular to that radius is in general finite.

An equation for the distances of the ends of this stretch from the origin is found, and expressions for the direction cosines given for the extreme rays which are at right angles to one another.

Various results concerning the lengths of perpendiculars are given; among them that the sum of the squares of the perpendiculars on three rays mutually at right angles to each other is constant.

The solid locus of the feet of the perpendiculars on the central axis generally is found to be the space between the sheets of the surface

$$\frac{x^2}{r^2-f^2} + \frac{y^2}{r^2-g^2} + \frac{z^2}{r^2-h^2} = 0 \quad . \quad . \qquad (E)$$

which is the reciprocal of the wave surface.

Lastly, the congruency of rays determined by (D), and the additional relation

$$\rho^4 + f^2\xi^2 + g^2\eta^2 + h^2\zeta^2 = f^4\lambda_1{}^2 + g^4\mu_1{}^2 + h^4\nu_1{}^2 \qquad (F)$$

is discussed, and shown to be of the fourth order. Minding's theorem is shown to hold when $f = 0$. (It is not true when $f \neq 0$). The equation to the surface locus of the feet of the perpendiculars on the rays of resultant is found, and so far as mere inspection goes is of the twelfth degree. In conclusion, the equations of various other loci connected with the congruency are given, or indicated to show the power of the methods employed.

Many of the above results were previously obtained quaternionically by Professor Tait. The interest of the present communication is less in the results obtained than in the methods employed to treat a particular problem in Plücker's "Line Geometry." In the development of the results Plücker's "Neue Geometrie" has been followed as far as possible. Any interested reader may be referred to that work for farther information on this and like matters.

3. Mathematical Notes. By Professor Tait.

(*a.*) On a Problem in Arrangements.

While making some algebraic problems last summer for an examination, I devised the following question:—

"A schoolmaster went mad, and amused himself by arranging the boys. He turned the dux boy down one place, the new dux two places, the next three, and so on till every boy's place had been altered at least once. Then he began again, and so on; till, after 306 turnings down, all the boys got back to their original places. This disgusted him, and he kicked one boy out. Then he was amazed to find that he had to operate 1120 times before all got back to their original places. How many boys were in the class?"

It is clear that one of the factors of the number of turnings down is $(n-1)$, where n is the number of boys in the class. The factors of 306 are 18 and 17, and those of 1120 are 7, 10, and 16. If we try 17 as the original value of $n-1$, 16 will be the value for one boy less: from which it appears by a tentative process that the class consisted of 18 boys. But it is interesting to examine the nature of the question more closely. It is intimately connected with one of the problems suggested in my paper on "Knots" (Trans. R.S.E., 1876–77, § 5). If we know the arrangement of the boys—after one of them has for the first time been turned to the foot of the class, the processes given in that paper lead easily to the complete solution.

Now it is easy to see that the particular arrangement just mentioned can be found diagrammatically as follows:—

Write down the numbers

1 2 1.

Put the double of the middle number to the right of it, and the next lower number to the left. Thus

1 3 2 4 1.

Operate in the same way on the numbers *last introduced*, and we have

1 5 3 6 2 7 4 8 1.

Continue in this way, and arrange these groups in successive order, leaving out the final 1 from each. We thus have the series

1, 2, 1, 3, 2, 4, 1, 5, 3, 6, 2, 7, 4, 8, 1, 9, 5, 10, 3, 11, 6, 12, 2, 13, 7, &c.

Strike off the first $n-1$ of these numbers (n being the number of boys), and the next n represent the arrangement of the class after all have been displaced: the numbers designating the several boys by their original places. Hence we have the key for translating the series into the successive derangements.

Another curious mode of getting this series is to begin with 1, then prefix 1, and insert 2, as below:—

1 2 1.

Again prefix 1, and insert 2, 3, 4, then

1 2 1 3 2 4 1,

and so on indefinitely.

It is worthy of remark that this series gives the integral of the equation

$$u_{2x+1} = u_x;$$

with the conditions

$$u_{2x} = x + 1,$$

$$u_1 = 1;$$

i.e., the solution of the following question:—

"Arrange an infinite row of numbers, those in the even places being 2, 3, 4, &c., so that if the first $(n-1)$ be struck off (n being any integer) the next n may consist of all the natural numbers from 1 to n inclusive."

Another result which these numbers present is the following:—

Every positive integer can be expressed, in one way only, by the sum of a finite number of terms of one of the infinite set of series

$$\begin{array}{c}
1 + 2 + 4 + 8 + 16 + \\
2 + 3 + 6 + 12 + 24 + \\
4 + 5 + 10 + 20 + 40 + \\
6 + 7 + 14 + 28 + 56 + \\
8 + 9 + 18 + 36 + 72 + \\
\&c., \&c.,
\end{array}$$

the partial sums for each being the several places occupied in the above series by each particular integer. This, however, is obvious when we consider that the sum of $(n+1)$ terms of any one of these series is of the form

$$(2r + 1)2^n - 1,$$

and that this expression can be made to equal any given positive integer by one definite pair (and one only) of values of r and n.

Thus we see that we may write

$$u_x = \tfrac{1}{2}(1 + \underline{x + 1}),$$

where the bar under $x+1$ means that it is to be divided by the highest power of 2 that it contains.

The numbers of operations, for classes of different numbers of boys from 2 to 25 inclusive, are in order as follows:—

2, 4, 9, 20, 30, 36, 28, 72, 36, 280, 110, 108, 182, 168, 75, 1120, 306, 432, 190, 140, 4410, 2772, 2530, 1440.

The calculation of the numerical value of any particular term is easy, but I have not attempted to express the general law of this very curious series. It seems, however, to be well worthy of attention, especially from the point of view of the expressions for numbers in the binary scale.

(*b.*) On a Graphical Solution of the Equation $V\rho\phi\rho = 0$.

This equation has been exhaustively treated in our Transactions by M. G. Plarr. The present note is a mere sketch of a graphical

solution. Let ϕ be divided into parts, one self-conjugate, the other not, then

$$\phi = \overline{\varpi} + V.\epsilon\,,$$

and the given equation may be written

$$\overline{\varpi}\rho + V\epsilon\rho = x\rho\,.$$

Hence $$S.\rho\left\{(\overline{\varpi} - x)\alpha + V\alpha\epsilon\right\} = 0\,.$$

whatever be α. Let α, β, γ be the principal unit-vectors of the pure strain $\overline{\varpi}$, and a, b, c (in descending order of magnitude) the associated scalars. Then the equation for x is, at once,

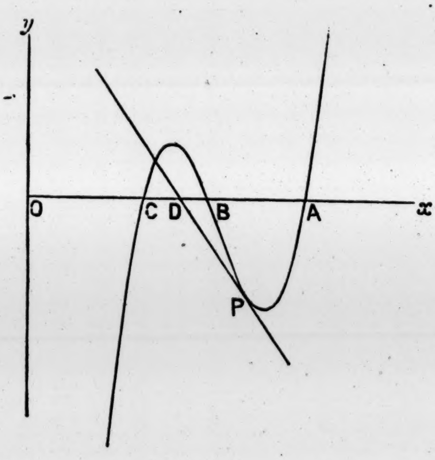
$$S.\Big((a - x)\alpha + V\alpha\epsilon\Big)\Big((b - x)\beta + V\beta\epsilon\Big)\Big((c - x)\gamma + V\gamma\epsilon\Big) = 0.$$

This may be written as

$$(x - a)(x - b)(x - c) - \epsilon^2(x + S.U\epsilon\,\overline{\varpi}U\epsilon) = 0\,.$$

Thus the problem is reduced to finding the limiting value of $T\epsilon$, for any given value of $U\epsilon$, so that the above equation may have all its roots real. This leads by the ordinary methods to a cubic in $T\epsilon^2$, but the expression is rather complicated.

For variety let us adopt a graphic method. It is obvious that the extreme values of $-S.U\epsilon\,\overline{\varpi}U\epsilon$ are a and c.



Let the curve represent the equation

$$y = (x - a)(x - b)(x - c),$$

and let OD represent any assumed value of $-S.U\epsilon\,\overline{\varpi}U\epsilon$. D must lie on the finite line AC. From D draw, as in the figure, a tangent

DP to the curve; and suppose a simple shear to be applied to the figure, parallel to the axis of y, so as to make this tangent coincide with the axis of x. The equation of the curve after the shear will obviously be

$$y = (x - a)(y - b)(z - c) + \tan \overset{<}{\mathrm{PDA}}\,(x - \mathrm{OD})$$

and it will touch the axis of x. Comparing this with the equation above, we see that we have for the maximum value required

$$T\epsilon^2 = \tan \overset{<}{\mathrm{PDA}}.$$

The absolute maximum of $T\epsilon$ is obviously when the point of contact is the point of inflexion of the curve (whose abscissa is $\frac{1}{3}(a+b+c)$), and the least values when D coincides with C or with A. These values are easily seen to be, in order,

$$\frac{a-c}{2}\sqrt{1+\tfrac{1}{3}\left(\frac{a-2b+c}{a-c}\right)^2},\ \frac{a-b}{2},\ \text{and}\ \frac{b-c}{2}.$$

BUSINESS.

Professor TURNER proposed the following motion, of which he gave notice at the General Statutory Meeting:—

"That the Honorary Vice-Presidents be in future members of the Council of the Royal Society, and that the Laws of the Society be modified to the extent necessary to carry this into effect."

This was seconded by Professor TAIT, and agreed to.

It was moved by Mr FERGUSON of Kinmundy, seconded by Professor DUNS, and agreed to—

"That it be remitted to the Council to bring up to the next Meeting the verbal alteration of Law XVII. required to carry out the above resolution."

Professor TURNER proposed the second resolution, of which he gave notice at the General Statutory Meeting, viz.:—

"That the Chairman of the Meetings of the Council should have a casting as well as a deliberative vote."

This was seconded by Professor TAIT, and agreed to.

Thomas Armstrong Elliot, M.A. Cantab., was balloted for, and declared duly elected a Fellow of the Society.

Monday, 19*th January* 1880.

Professor H. C. FLEEMING JENKIN, Vice-President, in the Chair.

The following Communications were read and Business transacted:—

1. Part of the Material employed by Principal Forbes in Tamping the Bore for his Earth Thermometers at the Edinburgh Observatory was exhibited in its altered state.

2. A New Method of investigating Relations between Functions of the Roots of an Equation, and its Coefficients. By J. D. H. Dickson, M.A.

3. Remarks on Mr Crookes's Recent Experiments. By Professor George Forbes.

The author explained that he was glad to be able to bring before the Society some apparatus illustrative of Dr Crookes's splendid researches on what he calls *radiant matter*. This he was able to do by the skill of Mr Gimingham, who had prepared some splendid vacuum tubes for him lately. He valued those researches very highly, because they brought most conclusive evidence to bear upon the truth of the kinetic theory of gases; because they gave us a clear insight into the action of the electric current and discharge; because they promise to help us in forming a notion of what electricity really is; and because we may also hope from these researches to get some knowledge of the nature of molecules.

It is now generally conceded that a gas consists of detached molecules, moving about with great rapidity, and rebounding from each other and from the sides of the containing vessel, with perfect elasticity; Clausius and Maxwell have shown that at any given pressure the molecules on an average can travel over only a certain

distance without experiencing a collision. In the case of hydrogen, at atmospheric pressure, this distance, called the *mean free path* of a molecule, is the $\frac{1}{250000}$th part of an inch. If the hydrogen in a tube at atmospheric pressure be gradually removed by an air-pump until only a small fraction of that quantity of gas remains in the tube, then each molecule can on an average traverse a distance much greater without having a collision with another molecule. During the time of passing over this space their vibrations are more or less given off to the other. Thus they soon cease to be luminous, even if incandescent when they last collided with a molecule. These facts explain the action of the radiometer.* They also explain the fact observed in *very* high vacua, that there is no luminosity round the negative pole† of a vacuum tube (highly exhausted) when two poles inside it are connected with the poles of an induction coil.

[The first experiment was here shown, when it was seen that in a very perfect vacuum no light was seen in the tube except a phosphorescence of the glass tube, due to the impulse of the molecules of gas upon it. When water-vapour was admitted by heating a subsidiary bulb containing caustic potash, the ordinary phenomena of vacuum tubes were gradually developed.]

The impact of these molecules on German glass or other phosphorescent substance produces intense phosphorescent light. [This was shown by placing in these vacuum tubes phosphorescent sulphides and rubies, and by directing the discharge upon German glass.]

When the electric density of a considerable portion of the negative electrode is uniform, we should be led to expect that the molecules of gas which strike it should be driven off perpendicular to the surface, that being the direction in which the repulsive action between the electrified electrode and the similarly electrified molecules should take place. This is exactly what happens. If the negative electrode be a circular disc, the molecules are driven off to the opposite side of the tube or bulb. This direction depends in no way upon the position of the positive pole. In this case the author remarked on the existence of a black mark in the centre of

* See Tait and Dewar, *Nature*, 1875. But also see Clerk Maxwell, R.S., 1879.

† The molecules of gas certainly fly off from the negative pole in obedience to the above laws. At present we know little about what happens at the positive pole.

the luminous phosphorescent patch produced on the glass by the molecular bombardment. When this is seen it is due to a stream of blue light from the centre of the negative disc. This stream is not deflected by a magnet as is the stream producing phosphorescence. Since the molecules are driven off perpendicular to the surface of the negative electrode, any object placed in their path shields the phosphorescent glass from the molecular bombardment, and thus a shadow of the object is thrown on the glass. The author drew attention to the fact that there was no penumbra to this shadow. Considerable heat is developed on the glass or any other object exposed to this molecular bombardment. If the negative electrode be a concave part of a sphere, the molecules are concentrated at the centre of the sphere, and produce there sufficient heat to melt platinum.

We know from the experiments of Rowland that a body charged with electricity, when in motion, acts like an electric current. So here these charged molecules in motion are deflected by a magnet as a current would be. But they do not yield to the electromagnetic force so much as a perfectly flexible linear conductor would do.

The author showed that the stream of molecules, when falling on glass, could be reflected on to a piece of mica covered with a phosphorescent powder; but he has not yet been able to determine whether they are reflected according to the laws of the reflection of light. He also drew attention to certain small black spots which he has occasionally noticed on the phosphorescing glass which move over it like bubbles, disappear, and are succeeded by others.

He suggested that efforts should be made to determine what happens to molecules at the positive pole—also to discover by what means the negatively electrified particles, after having been projected along the length of a tube, find their way back again, if not by regular reflection.

4. Additional Note on Minding's Theorem. By Professor Tait.

BUSINESS.

Professor Turner reported that in accordance with the remit made by the Society at last Ordinary Meeting, the Council recommended the following changes in the Laws of the Society:—

"That Law XVII be cancelled, and the following Law be established in its place:—

LAW XVII.

"That there shall be formed a Council consisting, first, of such gentlemen as may have filled the office of President, and, secondly of the following to be annually elected, viz., a President, six Vice-Presidents (two at least of whom shall be resident), twelve Ordinary Fellows as Councillors, a General Secretary, two Secretaries to the Ordinary Meetings, a Treasurer, and a Curator of the Museum and Library."

That the following addition be made to Law XV. :—

"The Council shall have power to regulate the private business of the Society. At any Meeting of the Council the Chairman shall have a casting as well as a deliberative vote."

Professor Turner moved, and Mr Sang seconded the adoption of these recommendations, which were agreed to.

The following Minute, prepared by the Committee appointed for the purpose by the Society at the General Statutory Meeting, was read and approved December 1, 1879. The Secretary was directed to send an extract to Dr Balfour:—

"The Members of the Royal Society of Edinburgh most cordially unite with the Council in acknowledging the long and valued services of Dr Balfour as General Secretary of the Society; they desire, as a body, to express how deeply they regret the loss of these services; and whilst sympathising with him in the circumstances which have led to the resignation of the office he has so long held in the Society, they would cherish the hope that he may enjoy in his retirement that serene repose which the consciousness of a well-spent life is fitted to insure."

The following letter in reply was now read:—

"Dear Professor Tait,—I feel highly honoured by the kind recognition, by the Fellows of the Royal Society, of my long

continued services as their Secretary, and I return my warmest thanks for the communication which you have transmitted to me.

"Be so good as convey my kind acknowledgment to the President and Fellows of the Royal Society.—I am, your obedient servant,

"J. H. Balfour."

Monday, 2d February 1880.

Professor DOUGLAS MACLAGAN, Vice-President, in the Chair.

The following Communications were read:—

1. On the Distribution of Temperature under the Ice in Frozen Lakes. By John Aitken.

In January and February 1879 Mr J. Y. Buchanan communicated to this society two papers, on the distribution of temperature under the ice in Linlithgow Loch. In these papers he gives a most interesting and valuable series of temperature observations made by him, of the water, at different points, and at different depths, in the loch while it was covered with ice. He also gives the temperatures as taken by him under similar circumstances in Loch Lomond. These observations by Mr Buchanan disclose a somewhat unexpected thermal condition of the water.

As water attains its maximum density at a temperature of 39·2° Fahr., it has generally been supposed that water in lakes ought never to fall below this temperature, except near the surface. Because when the water is cooled below 39·2° it will float over the hotter and denser water, and tend to keep the surface, while the latter water will tend to keep the bottom, and radiation and conduction will only enable a lower temperature than 39·2° to penetrate below the surface, very slowly, and to a very small depth. These theoretical expectations are, however, entirely upset by Mr Buchanan's temperature observations. He found that the greater part of the

water under the ice in Linlithgow Loch had only a temperature of about 37°, while in Loch Lomond the mean temperature was as low as 34°.

The object of the present communication is to offer an explanation of this unexpected condition of the water—to show how theory has failed correctly to predict the thermal conditions of a frozen lake. Mr Buchanan in the papers already referred to, and also in "Nature" of 6th March 1879, gives an explanation of how water in lakes becomes cooled below its temperature of maximum density. As this theory is at variance with many well-known results of experiment, and does not seem satisfactorily to explain the facts, it will be necessary first to show wherein this explanation fails. Mr Buchanan's theory is simply this:—The water in the lake gets gradually cooled to the temperature of its maximum density, namely 39·2°. Some time after all the water has acquired nearly this temperature, freezing begins at the edge first, while there is open water in the middle of the lake. The effect of this freezing would be expressed graphically by the dipping of the isothermal of 39·2° at the edge. This alteration of the temperature will be accompanied by an alteration in density; and if we consider a vertical section in the middle of the lake and another section at the edge, we will find the mean density at the middle greater than at the edge, the result of which is convection currents, flowing on the surface from the ice, and under currents from the middle towards the sides. This appears to be a perfectly correct statement of the condition of the water under the circumstances, but it does not seem capable of giving an explanation of the temperature of the water all through the lake being below 39·2°. The colder and lighter water near the edge, where freezing first begins, will certainly tend to spread itself towards the middle of the lake—but it will tend to keep the surface, and will not sink and produce a vertical circulation. And unless this cold surface water sinks how is the lower temperature to be carried downwards? We have here a precisely corresponding condition of matters to what exists in the lake on the return of summer, when the water is being heated. At first the temperature of the lake is below 39·2°, and the warmer water brought in by the rivers and that heated by the sun, sinks to the bottom and raises the mean temperature of the lake to the temperature of its maximum density. After it

has attained a temperature of 39·2°, the warmer inflowing water no longer sinks, but spreads itself over the colder water and keeps its place on the surface, and does not give rise to a vertical circulation, and carry its higher temperature down into the depths of the lake.

As theory might not have taken into consideration all the conditions which exist in a lake while freezing, an experiment was made, in which the conditions of a freezing lake were imitated as closely as possible, to see if Mr Buchanan's theory was correct, and to confirm or correct our previous knowledge. A trough 46 cm. long by 15 cm. broad and 12 cm. deep was filled with water. In the trough were fixed three thermometers, one was placed 1·5 cm. from the surface, one in the middle, and one 1·5 cm. from the bottom. The trough was made with glass sides, so that by dropping some coloured solution into the water, the direction, position, and velocity of the currents could be noted from time to time. Over the trough was placed a refrigerator, one end of which dipped into the trough and touched the water for a length of about 12 cm. This trough represented in miniature a section of a lake in which might be observed all the changes in currents and temperatures which take place due to the heating, cooling, and freezing of the water. Observations of temperature and currents were made and recorded from time to time during twenty-two hours. All these observations were in keeping with the theoretical knowledge we at present possess, and did not give support to the theory that the cooling of the water below 39·2° could give rise to convection currents which could carry their lower temperature into the depths of the lake.

The apparatus being arranged as described, the trough was filled with water at a temperature of 52·5° Fahr. A freezing mixture was put in the refrigerator; when this was done, currents at once began to flow. The surface current flowing *towards the cold*, that is towards the end at which the refrigerator touched the water, and the return current flowing near the bottom. These currents kept moving for two hours, during which time *all* the thermometers fell at very nearly the same rate, and at the end of the two hours they all indicated a temperature of about 41·5° The circulation during this time gradually got slower and slower, on account of the difference of density of the water on which the existence of these currents depended gradually diminishing.

A little later after the water had acquired nearly the temperature of its maximum density a change took place. In addition to the two currents described, another current began to set in, flowing on the surface and over the other two currents already described. Its direction being *from the cold*, that is in the opposite direction to the previous surface current. Its return current coming back with the previous surface current. This new surface current was produced by the water being cooled below its maximum density, and rising, instead of sinking as before. The effect of this new surface current was also indicated by the upper thermometer, which began to fall much faster than the other two. These two sets of currents kept flowing for some hours, gradually getting slower and slower, particularly the lower one, which seemed to stop after about four hours from the beginning of the experiment, by which time the water at the bottom had arrived at a temperature of about 40°. After eight hours almost all circulation had ceased even by the surface.

While the temperature of the water was above 39·2°, the heat was distributed by convection currents, causing all the water to circulate, and the temperature fell at about the *same rate at all depths*, and further it fell from 52·5° to 41·5°, or 11° in two hours. After the maximum density had been attained, and the cold current, instead of sinking to the bottom, flowed over the surface, the upper thermometer fell 7° in two hours, while the bottom thermometer during eight hours only fell 2°, being only 39·5 at the end of that time.

This experiment proves that the downward convection of heat ceased about the temperature of maximum density, as the lower thermometers ceased at about that temperature to fall at the same rate as the upper one. It is true the lower thermometer did fall below 39·2°, but it did so only very slowly, part of the fall being probably due to convection currents caused by the heat of the sides of the trough taking up the dense water, its place being supplied by the overlying colder water. It must also be remembered that the lower thermometer was only 10·5 cm. from the surface, so that the bottom water would lose part of its heat by radiation and conduction.

On removing the refrigerator and applying heat a reverse series of phenomena was observed. The hot water sunk and gave rise to a circulation, the surface current being *towards* the heat. The

return current was also near the surface, as the water was not heated to its maximum density it was unable to sink into the colder water at the bottom, and so flowed back over it and under the surface current. While this was taking place the upper thermometer was rising rapidly, while the lower ones did not show any signs of rising for some time. As the temperature of the surface water increased the return current got deeper and deeper, till it flowed at the bottom. The bottom thermometer then began to rise. This current continued to flow till the bottom water had acquired the temperature of maximum density, being some degrees below it previously. After all the water had acquired nearly the temperature of maximum density, a change took place. A new current began to flow on the surface *from* the heat, and over the previous surface current. The lower currents gradually became slower and slower, and at last stopped. After a time the surface current also ceased and all was still, the hot water resting quietly on the cold.

It is quite possible that the dipping of the isothermal of 39·2° referred to by Mr Buchanan, might give rise to a circulation of the water, but the conditions necessary for it to do so are not likely to happen, owing to the bad conducting power of water. The isothermal of 39·2° would require to dip to a great amount, to a number of feet *lower at the sides than in the middle of the lake*, in order to give rise to a sufficient "head" to put the water in motion. And even when a circulation is so produced, *the water flowing towards the cold region will not be drawn upwards from the bottom, but will flow in horizontally*, and will be water of a lower temperature than 39·2°; that is, the lower returning current will descend but little below the lowest point of the 39·2° isothermal, and all water at a lower level than this isothermal will be at rest, and retain its temperature of maximum density.

This point was illustrated in the experiment described. After ice had formed under the end of the refrigerator, and after the bottom water had acquired a temperature of about its maximum density, and was quite still, a current was noticed flowing slowly from the ice at the surface, and a return current immediately underneath; but more than a half of the water in the very shallow trough was unaffected by it, showing that the dense water does not rise from the bottom to keep up such a current, but that its supply is drawn from the

colder and lighter overlying water. After some hours even this surface current became so weak as not to be noticeable.

Convection currents being thus shown to be incapable of lowering the mean temperature of the water in lakes below 39·2°, I shall now call attention to a cause which, though not existing and not recognised in our laboratories, is yet in constant action in nature, and is quite sufficient to account for the thermal disturbance under consideration. The great vertical distributor of temperature in lakes is in all probability the winds which are constantly blowing over them. Previous to the freezing of the surface the winds were in constant action, producing currents in the water, which gave rise to the peculiar distribution of temperature we find after the lake is covered with ice. I have shown in a previous paper* that the great wind-driven currents of the ocean have but little influence on the vertical distribution of temperature of the oceanic waters. The circumstances are, however, quite different in lakes. In the ocean the wind only affects one part of the surface of the water, and gives rise to a circulation the greater part of which is *horizontal*, the return current coming back at a part of the ocean unaffected by the wind. But in a lake the wind blows in the same direction all over the lake, and the return current cannot come back on the surface, it is therefore compelled to sink and return *underneath* the surface wind-driven current. The cold surface water is thus compelled to sink and carry its low temperature into the depths of the lake. When we consider that water of a few degrees above and of a few degrees below 39·2° will have almost the same density, we will see that difference of density will oppose but little resistance to these wind-driven currents. And under almost all circumstances the "head" produced by the wind will be greater than that due to difference of density.

The temperatures as given by Mr Buchanan, so far as they go, are entirely in support of this wind theory. For instance, Linlithgow Loch is much smaller and better sheltered from the wind than Loch Lomond, and we find the mean temperature of Linlithgow is higher than Lomond, which we would quite expect, as it will have a less wind-driven circulation. Again, as Linlithgow Loch is smaller than Loch Lomond, the wind-driven surface currents will not be so deep, nor the return currents so near the bottom. We would therefore

* Proceedings of the Royal Society, Edinburgh, 1876–77.

Fig. 1.

Fig. 4.

Fig. 2.

Fig. 3.

From Photos.

McFarlane & Erskine, Lithrs. Edinr.

expect the water at the bottom of Linlithgow Loch to be warmer than the water at the bottom of Loch Lomond, which we find to be the case.

[*Added February* 24, 1880.—It is scarcely necessary to say that the explanation here given of the descent of the cold water into the depths of lakes in winter, applies to the converse process which takes place on the approach of summer. If the distribution of temperature in lakes depended on currents due to difference of temperature alone, the water in the depths of lakes would never rise above 39·2°, as the sun's rays are robbed of their heating effect by a small depth of water, —and it is principally by the wind-driven currents that the hotter and lighter surface water is caused to sink into the depths of the lake].

2. Remarks on the Aborigines of the Andaman Islands. By Surgeon E. S. Brander, M.B., C.M., H.M. Bengal Medical Staff, Second Medical Officer, Port Blair and Nicobars. (Plate XV.)

I propose making a few brief observations on the relations in which we stand to the aborigines of the Andaman Islands, their place in the economy of this convict settlement, and their more noticeable habits and customs. In the consideration of the latter, I shall only state such information as I have obtained myself from conversation with, and personal observation on the manners of the natives. Detailed monographs on their ethnology, &c., have been written already by several observers, and are to be found in the "Transactions of the Anthropological Society," &c.

When the British Government first acquired possession of these islands, the aborigines, due perhaps to previous bad treatment at the hands of Europeans, presented a decidedly hostile aspect. Any attempt to land from ships was met by determined resistance on their part, and for some years this condition of things remained. It was shortly after the Mutiny that Government first decided to use these islands as a convict settlement, deeming it a safe measure to get the imprisoned mutineers safely out of the country altogether. By the time the convict settlement was first established, the Andamanese had ceased any hostile efforts, but had entirely fled into the jungle, where they refused to accede to any friendly overtures. By

the judicious leaving of presents for them, and by the habitual kindness of reception shown to any of their number venturing into the settlement, this timidity was gradually overcome. At that time a zealous and devoted servant of the Government, Mr J. Homfray, voluntarily went into the jungle and lived amongst these savages as one of themselves. Ignoring the discomforts of the position and thoroughly ingratiating himself with them, he remained in their midst until he acquired a fluent knowledge of their imperfect but difficult dialect. By this means he established in them a confidence and friendly feeling towards the "strange white people." The policy of this course was soon apparent. These islands were destined to be a great convict settlement, and the co-operation and friendliness of the aborigines with the British Government would at once strengthen the hands of the latter in the management of the place, by rendering escapes practicably impossible.

At the present time all the tribes in the neighbourhood of Port Blair are on most friendly relationship with the Executive here. One of the settlement officers has it, as his special duty, to look after them. On one of the islands here, called Viper Island, there is a "Home" for the reception of such healthy members of the jungle tribes as like to come in. Here they are well housed and fed. In exchange they make bows, spears, and various rude ornaments, spear and store up turtle, &c., and by the sale of these augment the Government grant allowed for their support. There is also a school for Andamanese children on Ross Island (the chief station here), where they are taught to read, write, and speak English and Hindostani, together with the elements of arithmetic, &c., and the girls in addition, needlework. Besides this there are the Andamanese Hospital and the Convalescent House on Viper Island, which were under my direction when I was in the capacity of Additional Medical Officer. While in the latter function, I resided myself on Viper Island, and so had considerable opportunities of observing the Andamanese both in hospital and in their normal condition. Since my promotion to the duties of Second Medical Officer, their medical charge has unfortunately left my hands.

Finally, the Andamanese at present most efficiently fulfil the function of always having 20 to 50 armed men ready on the shortest notice, to scour the jungle and recover, dead or alive, any escaped

convicts. For each convict so recovered Government pays them five rupees. In the great majority of cases the runaways are easily recaptured without any resistance, being generally exhausted from exposure and want of food. Cases, however, have occurred where the escaped convicts have shown fight on being overtaken, and when the Andamanese have had to use their bows and arrows (of which I shall speak after) with rapidly fatal effect.

Such then, briefly, is the present relation in which these people stand to us, and their general function in the community. I may add that yearly they are becoming more tractable. Tribes from remoter parts of these islands are voluntarily sending in messengers; these are always well treated, and take away with them useful presents and a good opinion of their entertainers. In time, then, we may hope to stand in a friendly relation to all these tribes, though some still remain who refuse any attempts at conciliation.

I will not make any detailed remarks on the structural or other affinities of these people. These have been treated of in the detailed monographs to which I before referred; I will therefore at once proceed to my personal experiences. One physiognomical fact has appeared to me very noticeable, however, and that is, the remarkable diversity of the facial type. Some faces seem to resemble the Negraic, some the Malayan, and some even the Aryan in character. A reference to the accompanying photo-lithographic illustration (fig. 1) will help to render this clearer. Thus the "characteristic," *i.e.*, the most frequent, type of Andaman face would seem to be of a modified Negraic variety. I may add, however, that I have noticed less dissimilarity to exist between the women's faces than the men's. As an example of the variety of facial character observable, I would point out the man's face seen in profile in fig. 1, with its comparatively straight nose and compressed lips, as distinguished from the man's face (seen full face) next to him with the flat nose and thick protuberant lips. When a number of these people are seen together this peculiarity becomes very noticeable.

In size these people are remarkably small, and I append some tables of measurements I have made. From these it will appear that the average man does not exceed 4 feet 10 inches in height, with a weight of about 101 lbs.; and the average woman, 4 feet 6 inches, with a weight of about 98 lbs. From the figures in the table, I

find that the greatest height registered for a man was 5 feet 1½ inch, lowest height, 4 feet 7½ inches, giving a mean of 4 feet 10·43 inches for men's height.

The heaviest man measured was 117 lbs., the lightest 92, giving a mean of 101·4 lbs. for men's weight.

Men.				Women.			
No.	Weight in lbs.	Height.		No.	Weight in lbs.	Height.	
		Feet.	Inches.			Feet.	Inches.
1	117	5	...	1	94	4	5
2	112	4	11½	2	103	4	7
3	106	4	11½	3	96	4	7
4	106	4	11½	4	91	4	6
5	95	4	9½	5	93	4	5
6	100	4	10	6	102	4	4
7	92	4	8	7	89	4	5
8	96	4	10	8	85	4	4
9	97	4	8½	9	92	4	7
10	98	4	11	10	96	4	3½
11	94	4	8	11	126	4	9
12	90	4	7½	12	103	4	6
13	109	5	1½	13	120	4	8½
14	93	5	...	14	94	4	6½
15	114	5	...	15	96	4	7

Among the women the tallest was 4 feet 9 inches, and the shortest 4 feet 3½ inches; giving a mean height for them of 4 feet 6·03 inches. The heaviest woman weighed 126 lbs., and the lightest 85 lbs., giving a mean weight for these of 98·6 lbs.

The men are somewhat athletic and symmetrical in build, with fairly good chests and general muscular development.

The women are very ungainly; they undergo large development about the hips and abdomen. The latter becomes in any woman after 18 years of age very protuberant. This condition, I should think, is induced partly by the complete absence of any proper abdominal support during the period of pregnancy, and is partly due to the distension their stomachs undergo habitually after food. From childhood they cram their stomachs with an immense amount of bulky food in a short period, and they do literally "swell visibly" after their meals. This distended abdominal condition is noticeable in children of both sexes, but as the lads grow up they take more exercise, and their abdominal as well as other muscles become

firmer, and restrain the mechanical distension of the belly. With the women it is different; these latter influences do not exist, and when the child-bearing period comes the abdominal enlargement is augmented. The women also undergo exceptional enlargement in the gluteal region, the appearance of which is enhanced by their custom of attaching branches of twigs to their waist-bands, so as to hang over this region. This large development of abdomen, hip, and buttock, added to a very "waddling" style of gait, gives these women in progression a most grotesque appearance. They mostly possess considerable mammary development, and the glands in many instances seem to be in a chronic state of functional activity. This may be due to the late period to which they suckle their young (even to three and four years), or to another purpose to which the milk is applied, and to which I shall afterwards advert.

The Andamanese are strictly monogamous. The men marry at about eighteen years of age, and the women at any age after twelve, generally about thirteen or fourteen years. The number of children born to each pair is two or three, seldom more; with the women, from seventeen to twenty-five would seem to be the most fruitful period, and it is remarkable how aged the women become in appearance after having had one or two children. Labour with these women is a very simple business. They do their usual work until the actual commencement of the pains, and resume it very shortly after it is over. Two hours seems to be a fair allowance of time for an Andamanese woman to be delivered in.

The thinking powers of these people is not so much deficient as it is limited by their imperfect dialect. Thus, they have no conception of number beyond five, which they count by the fingers of one hand successively touching the nose. As a result they cannot express periods of time, and thus none of them know their own age. The latter can only be guessed at by their physical development and comparison with others of known age. Their ideas and dialect, in the crude state, are very imperfect. They can only reason about, and express ideas on the most tangible objects. Hence it is a matter of considerable difficulty to obtain their ideas on anything else, even by the aid of a fluent interpreter. Their ideas on things abstract are probably none, or are impossible to get at. On the above subject I have been careful to endeavour to get the views of the pure savage,

recently arrived from a remote part of the jungle, and having had no previous intercourse with white people. The Andamanese resident here are of no use for the purpose of ascertaining their primitive mental condition. They all have—especially the lads at school—undergone such a considerable mental enlargement, as I will instance in the case of a boy presently. The following brief statement of their views on some abstract ideas I obtained from a new arrival. Apparently some have no, and others the vaguest, conception of a deity or of departed spirits. By Pullôga they express the power which seems to have arranged natural objects as they now stand. They have a dim idea that some evil spirit or influence (no word to express that abstraction) sends disease among them, and that the latter is especially likely if they go into water after having rubbed on turtle fat! They have a vague fear of going about in the jungle when it is dark. This would seem to point to their recognition of some malign influence which is then liable to assail them. My informants could specialise no further on these matters, either from never having thought about the subject at all, or from want of words to answer such unusual inquiries.

In their dialect identical sounds would seem to have their meaning altered by the tone or key in which they are uttered. The result is that when speaking rapidly and consecutively, the voice modulates up and down in a manner not unlike a person speaking rapidly while humming a chant of three or four notes. The dialects of these islands are remarkable from their variety. The following are some:—

(1.) Bôjengîjîda, or South Andàman (spoken here).
(2.) Balawâda, or Andaman Archipelago.
(3.) Bôjigîâbda, or South Middle Andaman.
(4.) Âkâkôlda, or East Middle Andaman.
(5.) Awkojâwaida, or West Middle Andaman.
(6.) Âkâkêdê, or Interview Island.

Some of these are widely different, and the tribes living within twenty miles can often barely understand each other, those more remote being quite mutually unintelligible.

They are conservative in their language, and generally prefer to invent a new word for a previously unnamed object than to borrow it. A certain number of English and Hindostani words are now

used by them as "chirut" for cheeroot, "kâptan" for captain, "jômôdar" for "jemmadhar," a native policeman, &c.

As an instance of their inventive powers for new names for new objects, I may give—

Bîrmalâkâbangda, a steam-boat.

When asked the name of any, to them, strange objects, they readily extemporise one, as the other day when they were shown a thermometer and at once gave it a name! Doubtless, if the name they gave could be analysed, it would be found to mean "glass-stick," glittering ball, or some such tangible name, as they could form no approximate conception or idea of measuring heat.

As I before mentioned, these people are not deficient in brain-power; it rather lies dormant and unused in their savage state. When I had charge of the Andamanese Hospital, there was a patient in it called Jerry, an Andaman lad of twelve, who had been converted to Christianity and educated in the Ross school. He was of pure Andaman parentage, and had not attained the age of puberty, yet he could read English and Oordoo (Hindustani) fluently, as well as speak and write in both of these languages,—retaining also a knowledge of his Andaman language,—and had a fair knowledge of arithmetic. A number of these lads are being so educated, and the intelligence they display when the latter is developed is considerable.

By disposition these people are generous and affable, always merry and laughing. When one receives a present of tobacco, &c., he always shares it equally with his friends. The same is done with the proceeds of the chase. The men and women never work together. The men do little more than hunt pig, spear turtle, and catch fish. The women do all the remaining work of the community, including the bringing of firewood, and water, making and keeping up the fires, &c. The latter are also the barbers of the community, and it is in this capacity that their milk subserves the other use I before adverted to, viz., the function of shaving soap! It may be seen from the figures that these people all shave the head. When this process requires performing, the person to be shaved sits before the female barber, the latter then compresses her breast with the hand, and directing the nipple over the part to be shaved, emits a jet of milk. This fluid is rubbed in with the finger, and then the surface very cleanly shaved with a piece of broken glass or shell!

Their diet consists mostly of pig and turtle flesh, together with various edible roots, Jack fruit, plantains, mangoes, &c. Those in Hospital or in the Convalescent House have a regular daily ration of curry and rice, properly cooked. They also consume a large amount of sugar-cane, which they are very fond of chewing. I have seen a comparatively small child chew out the sugar from a yard of cane of an inch thick, with the result of visible distension of its stomach at the conclusion of the meal.

The men are good hunters and marksmen with the bow and arrow. In spearing turtle they use a barbed arrow with removable head. This comes out from the arrow shaft, but is attached by means of a strong cord to the boat containing the hunters; thus the turtles are secured alive and stored up in tanks. A similar plan is adopted in shooting pigs. They are very skilful at shooting fish under water, they seem intuitively to have calculated with great accuracy the difference of direction to be allowed from oblique aqueous refraction, and, I am told, shoot fish in this manner at a distance of thirty yards, with three-pronged barbed arrows.

Their bows are double curve bows of great strength. The bow strings are made by the women, out of the fibrous portion of certain jungle sapplings. This is obtained by picking off the outer bark and scraping it with a shell, which separates the fibrous portion, and the latter is subsequently made into cord by twisting between the hands. Their arrows are long and with variously-shaped heads, according to the use to which they are applied; some are seen in the plate (figs. 2 and 3). I have seen some arrows used in war, with such broad heads that they would completely disembowel an enemy if striking in the abdomen. They are not apparently acquainted with any substance for poisoning their shafts.

In personal adornment they are very curious, and their decorations seem, according to our ideas, very grotesque.

The men, when out hunting, &c., in the jungle wear little or no appendage. They paint and wear more ornaments when returned and living in their community, and of course most of all in connection with any "nautch" or public ceremony. The women are always more adorned. They, however, occasionally put on bracelets and anklets, consisting of simple bands with a number of curled leafy appendages attached thereto. In fig. 4 may be seen cer-

tain necklaces worn by the women mostly. These consist of the bones of their departed friends. The bones most generally used for this decorative purpose are the metacarpal, metatarsal, and the larger phalanges of hand and foot. I have seen a few necklaces in use where the vertebræ are used for this purpose. When the former class of bones are used they detach the articular extremities, and connect the medullary cavities by string as seen in the figure.

Another curious decoration is the attaching of a skull round the neck by string, and allowing it to hang down the back. This is only done with the skulls of chiefs or others who were of importance during their lifetime, and is generally reckoned a mark of esteem to the memory of the person whose skull is so used (fig. 3).

Tatooing the body is much practised by them, and is generally performed about the period of adolescence. It is performed by cutting the skin with sharp pieces of shell or glass, then washing the cut surface with water, then heating the same over a fire, until—I should imagine—it becomes painfully hot, and finally rubbing in a mixture of red clay and turtle fat. When quite healed, the cuts merely present the appearance of so many cicatrices, no especial coloration being retained by them. They also paint their bodies a great deal, and often in the most fantastic patterns, one side white checks, other side red stripes, &c. The colours mostly used are white, red, and a kind of yellow. The former painted all over the face as well as body, indicates that the person so adorned is in mourning for the recent decease of some near relative. The red paint is made by mixing a red (iron oxide) clay found here with turtle fat, and the yellow seems to be a mixture of this with the white, together with some coloured sap from one of the jungle trees. The paint is applied in all sorts of ways, a favourite one with the women being to paint the face white, with eyebrows and nose red!

As I before mentioned, they are a happy and joyous people, always in exuberant spirits. They are very fond of singing and dancing. When they perform a regular national dance the men are the dancers, the women and children forming the accompanying "band and chorus." The latter all sit in one place, and sing in approximate unison a series of notes, which, as far as they can be

represented in our musical notation, are indicated in the annexed staves—

This chant is accompanied by the rhythmical clapping of hands and beating of a resonant piece of wood, of which I have indicated the time in the separate notes above. This rhythmical clapping is kept up all through the performance, even when the singing stops. The dancers form in Indian file, the body slightly bent and the arms overhead, with the hands joined like in the position for diving. They then dance forward, one at a time, with a peculiar tripping step, giving the ground at every second pace a peculiar kind of back kick. The words of the chant are first given out solo by the leading dancer, and then taken up by the women's chorus, and repeated over and over again. They extemporise the words on any passing subject. Thus at one dance, at which I was present, I was the subject of their lay. The words they sang, broadly translated, having the following effect:—

> "Behold the Doctor protector [Mumjôla] (repeated).
> He gives medicine to our sick . . (repeated).
> Their wounds soon close . . . . (repeated).
> They return to hunt in the jungle" . (repeated).

Occasionally they repeat only the first line of the extemporised song.

When one of their number dies the body is taken into the jungle and placed in a small hut in a tree. There it is allowed to remain until the bones have become cleaned by the insects and weather. The bones are then used for ornaments as mentioned.

Explanation of Plate.

Fig. 1. Group of Andamanese men.
,, 2. Shows the mode of using the bow.
,, 3. Two Andamanese men—one wearing a skull on his back.
,, 4. Andamanese women with a bone necklace.

The figures are reproduced from photographs.

3. The Action of Sulphide of Potassium upon Chloroform. By W. W. J. Nicol, M.A. Communicated by Professor Crum Brown.

(*Abstract.*)

The author, after referring to the paper by Pfankuch,* who obtained by the action of sulphide of potassium upon chloroform a crystalline substance which he held to be a compound of sulphide of potassium and sulphoform, and to the paper by Bouchardat† on sulphoform, gives a detailed account of his own investigation.

The products formed when chloroform acts on an alcoholic solution of sulphide of potassium (prepared by dissolving caustic potash in alcohol, saturating one-half with sulphuretted hydrogen and then adding the other half) are sulphydrate of potassium and *thioformiate of potassium* (HCOSK, analogous in constitution to thiacetate). The action is probably as follows:—$HCCl_3 + 2K_2S = 3KCl + HCSSK$, and $HCSSK + H_2O = HCOSK + H_2S$, this sulphuretted hydrogen forming sulphydrate with the sulphide of potassium.

Thioformiate of potassium is converted into formiate by oxide of mercury. The silver salt blackens owing to formation of sulphide of silver, slowly at ordinary temperatures, rapidly on heating.

The free acid could not be prepared, its aqueous solution (prepared by the action of sulphuretted hydrogen on the lead salt suspended in water) rapidly decomposes, yielding formic acid.

4. Note on the Elimination of Linear and Vector Functions. By Professor Tait.

BUSINESS.

Professor Chrystal, George Ritchie Gilruth, L.R.C.S.E., D. Lloyd Roberts, F.R.C.P.L., A. H. Japp, LL.D., and Donald Ross, H.M. Inspector of Schools, were balloted for, and declared duly elected Fellows of the Society.

* Journal für pr. Chem. (2), 6, 99.

† Journal de Pharmacie, xxiii. 12.

Monday, 16*th February* 1880.

The Right Rev. BISHOP COTTERILL, Vice-President, in the Chair.

The following Communications were read:—

1. On the Geology of the Rocky Mountains. By Professor Geikie.

2. On Comets. By Professor Forbes.

The author commenced by stating that although these researches lead him to believe in the existence of two planets revolving in orbits external to that of Neptune, and although there was a great deal of evidence to show that he had actually determined the elements of the orbits, yet the latter point, being dependent on a coincidence of probabilities only, cannot be considered a certainty until the planets are observed.

The author accepts the theory of cometary orbits which supposes that these bodies, wandering through space, are attracted by the sun into the solar system so as to pursue parabolic orbits, and that some of these, in passing a planet, may have their velocity diminished, in which case they will afterwards describe an ellipse in a definite period.

It has long been known that the greatest distances (aphelion distances) to which comets recede from the sun are grouped into classes. Thus there is a large class of comets whose aphelion distance is about the same as the distance of Jupiter from the sun, and another large class with aphelion distance equal to Neptune's distance. The author has noticed that of the other periodic comets there is a large group of aphelion distances 100 times as far from the sun as the earth is, and another about 300 times. The rest of space is very free from aphelion distances. This is shown by the accompanying table.

At the British Association in 1879 Professor Newton of America proved some important propositions with respect to the introduction

of planets into the solar system. In answer to a question he said that his theory explained why the aphelion distance of a comet is generally about the same as the distance of the planet which rendered its orbit elliptic. The author then publicly stated that there could be no longer a doubt that two planets exist beyond the orbit of Neptune, one about 100 times, the other about 300 times the distance of the earth from the sun, with periods of revolution of about 1000 and 5000 years respectively.

If this be the case, the aphelion positions of a majority of the comets in each group would probably lie in one plane, which would be the plane of the planet's orbit. The analogy of the Jupiter group requires this; for although the orbits of the comets connected with Jupiter have every degree of inclination to the ecliptic, the aphelia of most of them are not far distant from the plane of Jupiter's orbit. Some, of course, might be expected to have been deflected from their original orbits slightly by planetary perturbations, especially those which had been a long time in the solar system.

The author calculated out the seven aphelion positions of comets which are grouped at a distance = 100, determining their latitude and longitude.

The author presented to himself the following problems:—1. Are there a fair number of these aphelion positions lying in one plane passing through the sun? 2. Determine the position of the nodes and inclination of this orbit. 3. Is it possible to imagine a planet, moving with tolerable uniformity, to occupy in the course of a few revolutions the aphelion positions exactly at the aphelion dates? 4. Is the velocity which we must for this reason assume about the same as that of a planet whose distance = 100? What is the present position of the planet? The seven comets of this group are:—

	Date.	Aphelion Distance.	Date of Aphelion.	Calculated Period.	L.
			A.D.	Years.	
I.	1840, iv.	96·7	1668	350	313°
II.	1843, i.	100·0	1655	376	225°
III.	1846	108·2	...	...	...
IV.	1861, i.	110·3	1654	413	139°
V.	1793, ii.	111·0	...	...	...
VI.	1861, ii.	111·2	...	...	...
VII.	1855, ii.	124·2	1608	493	192°

On marking upon a celestial globe the aphelion positions, it was immediately seen that four of them lay exactly upon a great circle, cutting the ecliptic at an angle of 53° and at the longitude 250°. This is the plane of the orbit of the supposed planet. It is remarkable that none of the four aphelion distances of the comets marked above I., II., IV., VII., diverge from this plane by more than 2 or 3 degrees.

The longitudes of these four aphelion distances measured on this plane of the hypothetical planet were then calculated; their values measured from ♌) are given in column L above. It is evidently impossible to suppose that the hypothetical planet could in one revolution, either by direct or retrograde motion, have passed each of the aphelion positions at the aphelion times. It was a matter of some labour to determine whether this could happen even in 2 or 3 revolutions, and a large number of hypotheses were tried. Eventually one was found agreeing well with facts, but involving suppositions as to previous apparitions of comets. According to this theory comet IV. was introduced into the system in 409 A.D., comet I. in 968, comet VII. in 1608, and comet II. in 1655. These are dates when the comets reached their aphelion. A planet moving at the rate of 2·788 years to 1°, or 997 years to a revolution, would, if it was 8° ahead of the comet IV. in 409 A.D., be 9° behind comet I. in 968, 6° behind comet VII. in 1608, and 8° ahead of comet VI. in 1651. These are all very close, provided we have evidence that comets IV. and I. have been seen so often before. Comet I. ought from the known period to have been visible in the years 1490 and 1140. The comet of the previous date has always been suspected to be the same as comet I. The comets which were seen about 1140 are so vaguely described that they cannot be identified.

Comet IV. ought to have been seen in 1861, 1446, 1031, 616 A.D., comets were seen in 1861, 1444, 1032, 617 A.D.

All apparitions previous to 1861 were seen about July, and in each case the star β Leonis is mentioned as being close to the comet. It remained only to see if the orbit of the comet IV. which appeared in 1861 suits these conditions. The author converted the heliocentric elements of the comet's position just before reaching the ecliptic into geocentric elements on the supposition of the earth being in its July position. The result is that the comet would be seen to pass within

a degree of β Leonis. The path would cut the ecliptic with geocentric longitude = 170°.

This will be conclusive evidence to all that these four comets are identical, and this in itself is an interesting deduction from the present research.

The eccentricity and longitude of perihelion of the plane of the planet's orbit can be estimated by noticing the different aphelion distances at different longitudes. This gives us ·08 as the eccentricity and ϖ about 290.°

The period of the supposed planet gives a mean distance = 98, which agrees with what goes before.

The elements of the planet for July 1880 are consequently—

$$a = 98^\circ$$
$$\Omega = 250^\circ$$
$$i = 53^\circ$$
$$\epsilon = \cdot 08^\circ$$
$$\varpi = 290^\circ$$

Motion direct.

Distance of planet from node = 296°.

Its present longitude measured from Ω up to the node and then along the planet's orbit is 185°.

[*Additional Note*, 31*st March* 1880.—Another calculation was made on the supposition of the planet being on the plane of the ecliptic, and affecting the comets when nearest to their aphelion positions. This gives its present longitude = 184°.

This hypothesis is strongly supported by the fact that the author has computed the position of Neptune by its influence on comets correct within 2°, although previously ignorant of its position.]

From the six comets whose aphelion distance is about 300 times the distance of the earth from the sun, the elements of the perturbing planet have roughly been calculated. This gives—

$$\Omega = 185^\circ$$
$$i = 45^\circ$$

nearly the same orbit as the preceding. The present position is—

R. A. 22h. 0m.
N. Declination 39°.

In the neighbourhood of this position, but at a position occupied by

the planet 100 years ago, was a star "11 Vulpeculæ," which Baily reported missing. The best orbit to suit the position of that star is—

$$\Omega = 262^\circ$$
$$i = 55^\circ.$$

If this be the true orbit the present position must be slightly altered.

Those comets which have been in the solar system for an enormous time would have their orbits probably deflected, and would not therefore appear in these calculations.

3. Note on the Velocity of Gaseous Particles at the Negative Pole of a Vacuum Tube. By Professor Tait.

The recent exhibition by Professor G. Forbes of some of the latest of Mr Crookes' experiments, together with what I had read or heard about their explanation, led me to infer that I might determine directly the velocity of the luminous particles near the negative pole (and perhaps at other parts) of a vacuum tube by means of observations of the spectrum made in directions perpendicular to, and parallel to, the lines of motion of the incandescent particles of gas.

I made the attempt on some charcoal-bromine vacuum tubes, for which I have to thank Professor Dewar, but I found the light to be so feeble that it was impossible to employ an eight-prism spectroscope. A one-prism spectroscope, when the spectra taken in and perpendicular to the direction of motion of the particles were placed side by side, showed merely that the velocity could not amount to anything like 90 miles per second. There did seem to be a very slight shifting of the former spectrum towards the violet, but this appearance was probably due to the fact that its light had been weakened by two reflections, while that of the other was taken direct.

It was evident that one cause at least of the failure is the great loss of light by multiplied reflections when a powerful spectroscope is employed. Thus I was driven to try the only other available method with which I was acquainted, and which indeed I had employed for more than ten years as an occasional part of the routine work in the physical laboratory. This method depends upon rotation (by quartz) of the plane of polarisation, combined (when necessary)

with sufficient prism dispersion just to separate the various bright lines of the source from one another.

My former assistant, Professor D. H. Marshall, made for me, in 1870, a series of careful measurements of the change of plane of polarisation of the lines C and F of hydrogen by this method, using a vacuum tube with a narrow bore, no slit, and a prism of small angle. It was found to give fair but not excellent results. Although no greater thickness of quartz was employed than the plates supplied along with Duboscq's saccharimeter, the planes of polarisation of C and F were separated by the thickest of them upwards of 130°; but the determination of the exact point of extinction is not easy. In measuring with practically homogeneous light, like that of a spirit-lamp with chloride of sodium or of lithium, the *prismatic* dispersion was, of course, not required. The great merit of the rotatory polarisation process consists in the fact that there is scarcely any additional loss of light incurred by using a foot or two of quartz instead of a few millimetres, and thus in proportion increasing the amount of rotatory displacement; while the thicker the quartz the less is the inevitable percentage error of observation. Also the position of each bright line is determined in terms of a standard quartz-rotation, and needs no comparison spectrum. It remains to be seen whether, on trial, it may be found possible to have a great length of quartz cut with sufficient accuracy, and whether the bright lines are narrow enough for this mode of observation. I have ordered a 6-inch cylinder of quartz, and hope soon to have observations made with it. Meanwhile it seems likely that this combination of polarising and analysing prisms, with a quartz plate, and a small direct vision spectroscope (with very wide slit), may be well adapted for measurements of position of the bright lines in the spectra of auroras, comets, and nebulæ, where it is not easy to employ either a comparison spectrum or a wire micrometer.

Monday, 1st March 1880.

PROFESSOR GEIKIE in the Chair.

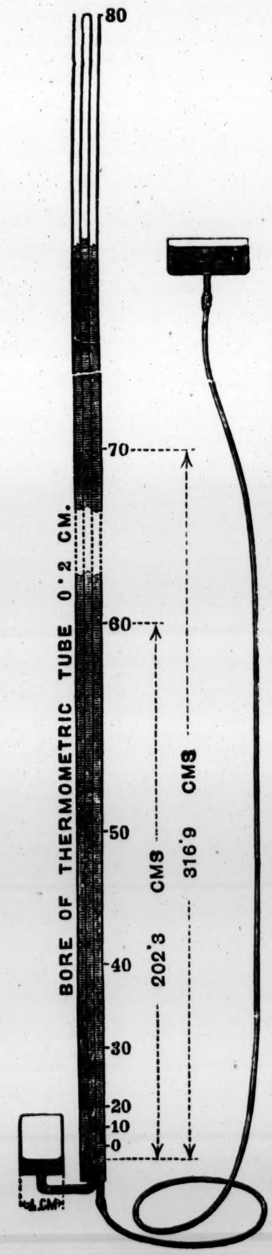


Fig. 1.

The following Communications were read:—

1. On Steam-Pressure Thermometers of Sulphurous Acid, Water, and Mercury. By Sir W. Thomson.

The first annexed diagram represents a thermometer constructed to show absolute temperature realised for the case of water and vapour of water as thermometric substance. The containing vessel consists of a tube with cylindric bulb like an ordinary thermometer; but, unlike an ordinary thermometer, the tube is bent in the manner shown in the drawing. The tube may be of from 1 to 2 or 3 millims. bore, and the cylindrical part of the bulb of about ten times as much. The length of the cylindrical part of the bulb may be rather more than $\frac{1}{100}$ of the length of the straight part of the tube. The contents, water and vapour of water, are to be put in and the glass hermetically sealed to enclose them, with the utmost precautions to obtain pure water as thoroughly freed from air as possible, after better than the best manner of instrument makers in making cryophoruses and water hammers. The quantity of water left in at the sealing must be enough to fill the cylindrical part of the bulb and the horizontal branch of the tube. When in use the straight part of the

tube must be vertical with its closed end up, and the part of it occupied by the manometric water-column must be kept at a nearly enough definite temperature by a surrounding glass jacket-tube of ice-water. This glass jacket-tube is wide enough to allow little lumps of ice to be dropped into it from its upper end, which is open. By aid of an india-rubber tube connected with its lower end, and a little movable cistern, as shown in the drawing, the level of the water in the jacket is kept from a few inches above to a quarter of an inch below that of the interior manometric column. Thus, by dropping in lumps of ice so as always to keep some unmelted ice floating in the water of the jacket, it is easy to keep the temperature of the top of the manometric water-column exactly at the freezing temperature. As we shall see presently, the manometric water below its free surface may be at any temperature from freezing to 10° C. above freezing without more than $\frac{1}{40}$ per cent. of hydrostatic error. The temperature in the vapour-space above the liquid column may be either freezing or anything higher. It ought not to be lower than freezing, because, if it were so, vapour would condense as hoar frost on the glass, and evaporation from the top of the liquid column would either cryophoruswise freeze the liquid there, or cool it below the freezing point.

The chief object of keeping the top of the manometric column exactly at the freezing-point is to render perfectly definite and constant the steam-pressure in the space above it.

A second object of considerable importance when the bore of the tube is so small as one millimetre, is to give constancy to the capillary tension of the surface of the water. The elevation by capillary attraction of ice-cold water in a tube of one millimetre bore is about 7 millims. The constancy of temperature provided by the surrounding iced water will be more than sufficient to prevent any perceptible error due to inequality of this effect. To avoid error from capillary attraction the bore of the tube ought to be very uniform, if it is so small as one millimetre. If it be three millimetres or more, a very rough approach to uniformity would suffice.

A third object of the iced-water jacket, and one of much more importance than the second, is to give accuracy to the hydrostatic measurement by keeping the density of the water throughout the long vertical branch definite and constant. But the density of water

at the freezing point is only $\frac{1}{40}$ per cent. less than the maximum density, and is the same as the density at 8° C.; and therefore when $\frac{1}{40}$ per cent. is an admissible error on our thermometric pressure, the density will be nearly enough constant with any temperature from 0° to 10° C. throughout the column. But on account of the first object mentioned above, the very top of the water-column must be kept with exceeding exactness at the freezing temperature.

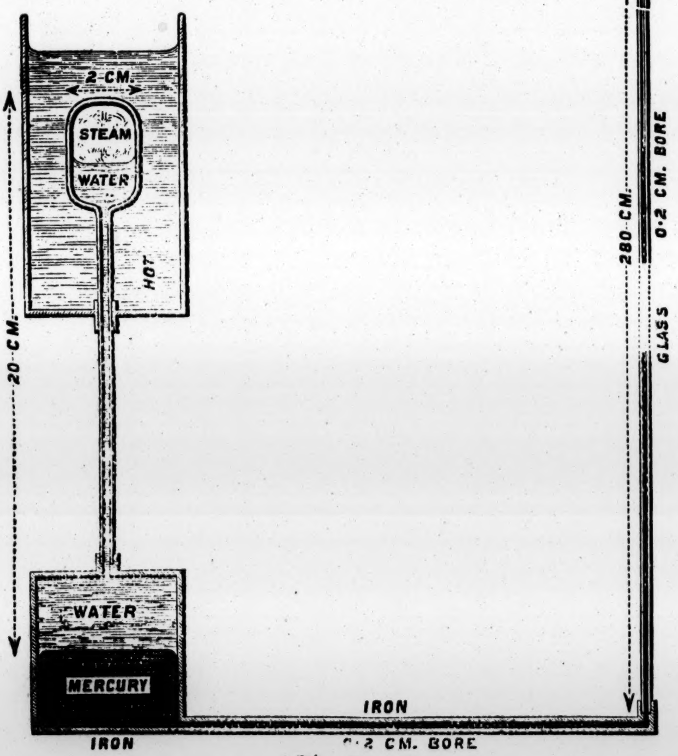


Fig. 2.

In this instrument the "thermometric substance" is the water and vapour of water in the bulb, or more properly speaking the portions of water and vapour of water infinitely near their separating interface. The rest of the water is merely a means of measuring hydrostatically the fluid pressure at the interface. When the temperature is so high as to make the pressure too great to be conveniently measured by a water column, the hydrostatic measurement may be done, as shown in the second annexed drawing (fig. 2), by a mercury column in a glass tube, surrounded by a glass water jacket not shown in the drawing, to keep it very accurately at some

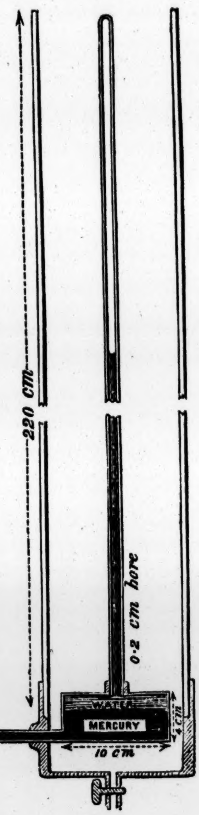
definite temperature so that the density of the mercury may be accurately known.

The simple form of steam thermometer represented with figured dimensions in the first diagram will be very convenient for practical use for temperatures from freezing to 60°. Through this range the pressure of vapour of water, reckoned in terms of the balancing column of water of maximum density, increases from $6\frac{1}{4}$ to 202·4 centimetres; and for this, therefore, a tube of a little more than 2 metres will suffice. From 60° to 140° the pressure of steam now reckoned in terms of the length of a balancing column of mercury at 0° increases from 14·88 to 271·8 centimetres; and for this a tube of 280 centimetres may be provided. For higher temperatures a longer column, or several columns, as in the multiple manometer, or an accurate air pressure-gauge, or some other means, such as a very accurate instrument constructed on the principle of Bourdon's metallic pressure-gauge, may be employed, so as to allow us still to use water and vapour of water as thermometric substance.

High-pressure Steam Thermometer.

At 230° C., the superior limit of Regnault's high-pressure steam experiments, the pressure is 27·53 atmos, but there is no need for limiting our steam thermometer to this temperature and pressure. Suitable means can easily be found for measuring with all needful accuracy much higher pressures than 27 atmos. But at so high a temperature as 140°, vapour of mercury measured by a water column, as shown in the diagram (fig. 3), becomes available for purposes for which one millimetre to the degree is a sufficient sensibility. The mercury-steam-pressure thermometer, with pressure measured by water-column, of dimensions shown in the drawing, serves from 140° to 280° C., and will have very ample sensibility through the upper half of its scale. At 280° its sensibility will be about $4\frac{3}{4}$ centimetres to the degree! For temperatures above 280° sufficient sensibility for most purposes is obtained by substituting mercury for water in that simplest form of steam thermometer shown in fig. 1, in which the pressure of the steam is measured by a column of the liquid itself kept at a definite temperature. When the liquid is mercury there is no virtue in the parti-

cular temperature 0° C., and a stream of water as nearly as may be of atmospheric temperature will be the easiest as well as the most accurate way of keeping the mercury at a definite temperature. As the pressure of mercury steam is at all ordinary atmospheric temperatures quite imperceptible to the hydrostatic test when mercury itself is the balancing liquid, that which was the chief reason for fixing the temperature at the interface between liquid and vapour at the top of the pressure-measuring column when the balancing liquid was water, has no weight in the present case; but, on the other hand, a much more precise definiteness than the ten degrees latitude allowed in the former case for the temperature of the main length of the manometric column



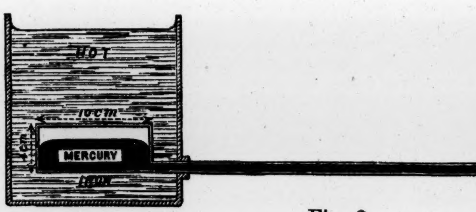


Fig. 3.

is now necessary. In fact, a change of temperature of 2·2° in mercury at any atmospheric temperature produces about the same proportionate change of density as is produced in water by a change of temperature from 0° to 10°, that is to say, about $\frac{1}{20}$ per cent.; but there is no difficulty in keeping, by means of a water jacket, the mercury column constant to some definite temperature within a vastly smaller margin of error than 2·2°, especially if we choose for the definite temperature something near the atmospheric temperature at the time, or the temperature of whatever abundant water supply may be available. If the glass tube for the pressure-measuring mercury column be 838 centimeters long, the simple mercury-steam thermometer may be used up to 520° C., the highest temperature reached by Regnault in his experiments on mercury-steam. By using an iron bulb and tube for the part of the thermometer exposed to the high

temperature, and for the lower part of the measuring column to within a few metres of its top, with glass for the upper part to allow the mercury to be seen, a mercury-steam-pressure thermometer can with great ease be made which shall be applicable for temperatures giving pressures up to as many atmospheres as can be measured by the vertical height available. The apparatus may of course be simplified by dispensing with the Torricellian vacuum at the upper end of the tube, and opening the tube to the atmosphere, when the steam-pressure to be measured is so great that a rough and easy barometer observation gives with sufficient accuracy the air-pressure at the top of the measuring column. The easiest, and not necessarily in practice the least accurate, way of measuring very high pressures of mercury-steam will be by enclosing some air above the cool, pressure-measuring column of mercury, and so making it into a compressed-air pressure-gauge, it being understood that the law of compression of the air under the pressures for which it is to be used in the gauge is known by accurate independent experiments such as those of Regnault on the compressibility of air and other gases.

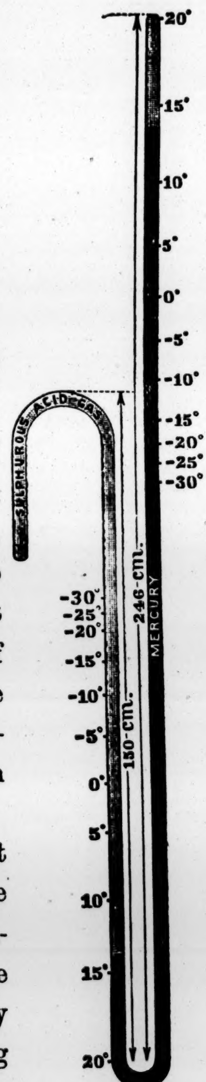


Fig. 4.

The water-steam thermometer may be used, but somewhat precariously, for temperatures below the freezing-point, because water, especially when enclosed and protected as the portion of it in the bulb of our thermometer is, may be cooled many degrees below its freezing-point without becoming frozen; but, not to speak of the uncertainty or instability of this peculiar condition of water, the instrument would be unsatisfactory on account of insufficient thermometric sensibility for temperatures more than two or three degrees below the freezing-point. Hence, to make a steam thermometer for such temperatures some other substance than water should be taken, and none seems better adapted for the purpose than sulphurous acid, which, in the apparatus represented with

figured dimensions in the accompanying diagram (fig. 4), makes an admirably convenient and sensitive thermometer for temperatures from $+20°$ to something far below $-30°$, as we see from the results of Regnault's measurements.

To sum up, we have, in the preceding description and drawings, a complete series of steam-pressure thermometers, of sulphurous acid, of water, and of mercury, adapted to give absolutely definite and highly sensitive thermometric indications throughout the wide range from something much below $-30°$ to considerably above $520°$ of the centigrade scale. The graduation of the scales of these thermometers to show absolute temperature is to be made by calculation from the thermodynamic formula—

$$t = t_0 \epsilon^{\int_{p_0}^{p} \frac{(1-\sigma)dp}{J\rho\kappa}}$$

where t denotes the absolute temperature corresponding to steam-pressure p; p_0 to the absolute temperature corresponding to steam-pressure p_0; κ the latent heat of the steam per unit mass; ρ the density of the steam; and σ the ratio of the density of the steam to the density of the liquid in contact with it. When the requisite experimental data, that is to say, the values of σ and $\rho\kappa$ for different values of p throughout the range for which each substance is to be used as thermometric fluid are available, the graduation of the scales of these thermometers to show absolute temperature can be performed in practice by calculation from the formula. Hitherto these requisites have not been all given by direct experiment for any one of the three substances with sufficient accuracy for our thermometric purpose through any range whatever. Water, naturally, is the one for which the nearest approach to the requisite information has been obtained. For it Regnault's experiments have given, no doubt with great accuracy, the values of p and of κ for all temperatures reckoned by his normal air thermometer, which we now regard merely as an arbitrary scale of temperature, through the range from $-30°$ to $+230°$. If he, or any other experimenter, had given us with similar accuracy through the same range the values of ρ and σ for temperatures reckoned on the same arbitrary scale, we should have all the data from experiment required for the graduation of our water-steam thermometer to absolute thermodynamic scale.

For it is to be remarked that all reckoning of temperature is eliminated from the second member of the formula, and that, in our use of it, Regnault's normal thermometer has merely been referred to for the values of $\rho\kappa$ and of $1 - \sigma$, which correspond to stated values of p. The arbitrary constant of integration, t_0, is truly arbitrary. It will be convenient to give it such a value that the difference of values of t between the freezing-point of water and the temperature for which p is equal to one atmo shall be 100, as this makes it agree with the centigrade scale in respect to the difference between the numbers measuring the temperatures which on the centigrade scale are marked 0° and 100°. Indirectly, by means of experiments on hydrogen gas, this assignation of the arbitrary constant of integration would give 273 for the absolute temperature 0° C., and 373 for that of 100° C, as is proved in p. 56 of the article on "Heat," in the *Encyclopædia Britannica.* Meantime, as said above, we have not the complete data from direct experiments even on water-steam for graduating the water-steam thermometer; but, on the other hand we have, from experiments on air and on hydrogen and other gases, data which allow us to graduate indirectly any continuous intrinsic thermoscope according to the absolute scale. By thus indirectly graduating the water-steam thermometer, we learn the density of steam at different temperatures with more probable accuracy than it has hitherto been made known by any direct experiments on water-steam itself.

Merely viewed as a continuous intrinsic thermoscope, the steam-thermometer, in one or other of the forms described above to suit different parts of the entire range from the lowest temperatures to temperatures somewhat above 520°, is no doubt superior in the conditions requisite for accuracy to every other thermoscope of any of the different kinds hitherto in use; and it may be trusted more surely for accuracy than any other as a thermometric standard when once it has been graduated according to the absolute scale, whether by practical experiments on steam, or indirectly by experiments on air or other gases. In fact, the use of steam-pressure measured in definite units of pressure, as a thermoscopic effect, in the steam thermometer is simply a continuous extension to every temperature, of the principle already practically adopted for fixing the temperature which is called 100° on the centigrade scale; and

it stands on precisely the same theoretical footing as an air thermometer, or a mercury-in-glass thermometer, or an alcohol thermometer, or a methyl-butyrate thermometer, in respect to the graduation of its scale according to absolute temperature. Any one intrinsic thermoscope may be so graduated ideally by thermodynamic experiments on the substance itself without the aid of any other thermometer or any other thermometric substance; but the steam-pressure thermometer has the great practical advantage over all others, except the air thermometer, that these experiments are easily realisable with great accuracy instead of being, though ideally possible, hardly to be considered possible as a practical means of attaining to thermodynamic thermometry. In fact, for water-steam it is only the most easily obtained of experimental data, the measurement of the density of the steam at different pressures, that has not already been actually obtained by direct experiment. Whether or not when this lacuna has been filled up by direct experiments, the data from water-steam alone may yield more accurate thermodynamic thermometry than we have at present from the hydrogen or nitrogen gas thermometer,—to be described in a subsequent communication to the Royal Society (Proceedings, April 19, 1880)—we are unable at present to judge. But when once we have the means, directly from itself, or indirectly from comparison with hydrogen or nitrogen or air thermometers, of graduating once for all a sulphurous acid steam thermometer, water-steam thermometer, or mercury-steam thermometer, that is to say, when once we have a table of the absolute thermodynamic temperatures corresponding to the different steam-pressures of the substances sulphurous acid, water, and mercury, we have a much more accurate and more easily reproducible standard than either the air or gas thermometer of any form, or the mercury thermometer, or any liquid thermometer can give. In fact, the series of steam thermometers for the whole range from the lowest temperatures can be reproduced with the greatest ease in any part of the world by a person commencing with no other material than a piece of sulphur and air to burn it in,[1] some pure water, some pure mercury, and

[1] Practically, the best ordinary chemical means of preparing sulphurous acid, as from sulphuric acid, by heating with copper, might be adopted in preference to burning sulphur.

abundance of ice and salt to make freezing mixtures; and with no other apparatus than can be made by a moderately skilled glass-blower; and with no other standard of physical measurement of any kind than an accurate linear measure. He may assume the force of gravity to be that calculated for his latitude with the ordinary rough allowance for his elevation above the sea, and his omission to measure with higher accuracy the actual force of gravity in his locality can lead him into no thermometric error which is not incomparably less than the inevitable errors in the reproduction and use of the air thermometer, or of mercury or other liquid thermometers. In temperatures above the highest for which mercury-steam pressure is not too great to be practically available, nothing hitherto invented but Deville's air thermometer with hard porcelain bulb suited to resist the high temperature is available for accurate thermometry.

The following statement is in the *Encyclopædia Britannica* article "Heat," appended to the description of steam-pressure thermometers which it contains:—"We have given the steam thermometer as our first example of thermodynamic thermometry because intelligence in thermodynamics has been hitherto much retarded, and the student unnecessarily perplexed, and a mere quicksand has been given as a foundation for thermometry, by building from the beginning on an ideal substance called perfect gas, with none of its properties realised rigorously by any real substance, and with some of them unknown, and utterly unassignable, even by guess. But after having been moved by this reason to give the steam-pressure thermometer as our first theoretical example, we have been led into the preceding carefully detailed examination of its practical qualities, and we have thus become convinced that though hitherto used in scientific investigations only for fixing the "boiling-point," and (through an inevitable natural selection) by practical engineers for knowing the temperatures of their boilers by the pressures indicated by the Bourdon's gauge, it is destined to be of great service both in the strictest scientific thermometry, and as a practical thermometer for a great variety of useful applications."

2. On a Sulphurous Acid Cryophorus. By Sir W. Thomson.

(*Abstract.*)

The instrument exhibited to the Royal Society consisted of a U-shaped glass tube stopped at both ends, containing sulphurous acid liquid and steam. The process by which the sulphurous acid is freed from air, which was partially exhibited to the Royal Society, is as follows:—

Begin with a glass U tube open at both ends, and attach to each a small convenient, very fine, and perfectly gas-tight, stop-cock. Placing it with the bend down in a freezing mixture, condense pure well-dried sulphurous acid gas direct into it from the generator till it is full nearly to the tops of the two branches. Then close the stop-cock, detach from the generator, and remove from the freezing mixture. Holding it still with the bend down, apply gentle heat to the bend, by a warm hand or by aid of a spirit-lamp, so as to produce boiling, the bubbles rising up in either one or the other of the two branches. After doing this for some time let the bend cool, and apply gentle heat to the surface of the liquid in that one of the branches into which the bubbles passed. With great care now open very slightly the stop-cock at the top of this branch, until the liquid is up to very near the top of the tube, and close the stop-cock before it begins to blow out. Repeat the process several times, causing the bubbles sometimes to rise up one branch, and sometimes up the other. After this has been done two or three dozen times, it is quite certain that only a very infinitesimal amount of air can have remained in the apparatus. When satisfied that this is the case, sink the bend once more into a freezing mixture, and with a convenient blow-pipe and flame melt the glass tube below each stop-cock so as to hermetically seal the two ends of the U tube, and detach them from the stop-cocks. This completes the construction of the sulphurous acid cryophorus.

The instrument, if turned with the bend up and the two sealed ends down, may be used as a cryophorus presenting interesting peculiarities.

The most interesting qualities are those which it presents when

held with the bend down. In this position it constitutes a differential thermometer of exceedingly high sensibility, founded on the difference of sulphurous acid steam-pressure due to difference of pressure in the two branches. One very remarkable and interesting feature is the exceeding sluggishness with which the liquid finds its level in the two branches when the external temperature is absolutely uniform all round. In this respect it presents a most remarkable contrast with a U tube, in other respects similar, but occupied by water and water-steam instead of sulphurous acid and sulphurous-acid-steam. If the U tube of water be suddenly inclined 10 or 20 degrees to the vertical in the plane of the two branches, the water oscillates before it settles with the free surfaces in the two branches at the same level. When the same is done to the U tube of sulphurous acid, it seems to take no notice of gravity; but in the course of several minutes it is seen that the liquid is sinking slowly in one branch and rising in the other towards identity of level. The reason is obvious.

3. Vibrations of a Columnar Vortex.

By Sir William Thomson.

This is a case of fluid motion, in which the stream lines are approximately circles, with their centres in one line (the axis of the vortex) and the velocities approximately constant, and approximately equal at equal distances from the axis. As a preliminary to treating it, it is convenient to express the equations of motion of a homogeneous incompressible inviscid fluid (the description of fluid to which the present investigation is confined) in terms of "columnar co-ordinates" r, θ, z, that is co-ordinates such that $r \cos \theta = x$, $r \sin \theta = y$.

If we call the density unity, and if we denote by $\dot{x}, \dot{y}, \dot{z}$ the velocity-components of the fluid particle which at time t is passing through the point (x, y, z); and by $\frac{d}{dt}, \frac{d}{dx}, \frac{d}{dy}, \frac{d'}{dz}$ differentiations respectively on the supposition of x, y, z constant, t, y, z constant, t, x, z constant, and t, x, y constant, the ordinary equations of motion are

$$\left.\begin{aligned}
-\frac{dp}{dx} &= \frac{d\dot{x}}{dt} + \dot{x}\frac{d\dot{x}}{dx} + \dot{y}\frac{d\dot{x}}{dy} + \dot{z}\frac{d\dot{x}}{dz} \\
-\frac{dp}{dy} &= \frac{d\dot{y}}{dt} + \dot{x}\frac{d\dot{y}}{dx} + \dot{y}\frac{d\dot{y}}{dy} + \dot{z}\frac{d\dot{y}}{dz} \\
-\frac{dp}{dz} &= \frac{d\dot{z}}{dt} + \dot{x}\frac{d\dot{z}}{dx} + \dot{y}\frac{d\dot{z}}{dy} + \dot{z}\frac{d\dot{z}}{dz}
\end{aligned}\right\} \quad . \quad . \quad (1),$$

and

$$\frac{d\dot{x}}{dx} + \frac{d\dot{y}}{dy} + \frac{d\dot{z}}{dz} = 0 \quad . \quad . \quad . \quad (2).$$

To transform to the columnar co-ordinates we have

$$\left.\begin{aligned}
x &= r\cos\theta,\ y = r\sin\theta \\
\dot{x} &= \dot{r}\cos\theta - r\dot{\theta}\sin\theta \\
\dot{y} &= \dot{r}\sin\theta + r\dot{\theta}\cos\theta \\
\frac{d}{dx} &= \cos\theta\frac{d}{dr} - \sin\theta\frac{d}{rd\theta} \\
\frac{d}{dy} &= \sin\theta\frac{d}{dr} + \cos\theta\frac{d}{rd\theta}
\end{aligned}\right\} \quad . \quad . \quad (3),$$

The transformed equations are

$$\left.\begin{aligned}
-\frac{dp}{dr} &= \frac{d\dot{r}}{dt} + \dot{r}\frac{d\dot{r}}{dr} - \frac{(r\dot{\theta})^2}{r} + \dot{\theta}\frac{d\dot{r}}{d\theta} + \dot{z}\frac{d\dot{r}}{dz} \\
-\frac{dp}{rd\theta} &= r\frac{d\dot{\theta}}{dt} + \dot{r}\frac{d(r\dot{\theta})}{dr} + \dot{r}\dot{\theta} + \dot{\theta}\frac{d(r\dot{\theta})}{d\theta} + \dot{z}\frac{d(r\dot{\theta})}{dz} \\
-\frac{dp}{dz} &= \frac{d\dot{z}}{dt} + \dot{r}\frac{d\dot{z}}{dr} + \dot{\theta}\frac{d\dot{z}}{d\theta} + \dot{z}\frac{d\dot{z}}{dz}
\end{aligned}\right\} \quad . \quad (4),$$

and

$$\frac{d\dot{r}}{dr} + \frac{\dot{r}}{r} + \frac{d(r\dot{\theta})}{rdr} + \frac{d\dot{z}}{dz} = 0 \quad . \quad . \quad . \quad (5).$$

Now let the motion be approximately in circles round $0z$, with velocity everywhere approximately equal to T, a function of r; and to fulfil these conditions assume

$$\left.\begin{aligned}
&\dot{r} = \varrho\cos mz\sin(nt - i\theta);\quad r\dot{\theta} = \mathrm{T} + \mathfrak{r}\cos mz\cos(nt - i\theta) \\
&\dot{z} = w\sin mz\sin(nt - i\theta);\quad p = \mathrm{P} + \varpi\cos mz\cos(nt - i\theta) \\
&\qquad\qquad \text{with } \mathrm{P} = \int\frac{\mathrm{T}^2 dr}{r}
\end{aligned}\right\} \quad (6);$$

where ϱ, $\mathfrak{r}$, w, and ϖ are functions of r, each infinitely small, in

comparison with T. Substituting in (4) and (5) and neglecting squares and products of the infinitely small quantities we find,

$$\left.\begin{aligned} -\frac{d\varpi}{dr} &= \left(n - i\frac{\mathrm{T}}{r}\right)\varrho - 2\frac{\mathrm{T}}{r}\tau \\ -\frac{i\varpi}{r} &= -\left(n - i\frac{\mathrm{T}}{r}\right)\tau + \left(\frac{\mathrm{T}}{r} + \frac{d\mathrm{T}}{dr}\right)\varrho \\ +m\varpi &= \left(n - i\frac{\mathrm{T}}{r}\right)w \end{aligned}\right\} \quad . \quad . \quad (7),$$

$$\frac{d\varrho}{dr} + \frac{\varrho}{r} + \frac{i\tau}{r} + mw = 0 \quad . \quad . \quad . \quad . \quad . \quad (8).$$

Taking (7), eliminating ϖ, and resolving for ϱ, τ, we find

$$\left.\begin{aligned} \varrho &= \frac{1}{m\mathrm{D}}\left(n - i\frac{\mathrm{T}}{r}\right)\left\{\left(n - i\frac{\mathrm{T}}{r}\right)\frac{dw}{dr} - \frac{i}{r}\left(\frac{\mathrm{T}}{r} + \frac{d\mathrm{T}}{dr}\right)w\right\} \\ \tau &= \frac{1}{m\mathrm{D}}\left\{\left(\frac{\mathrm{T}}{r} + \frac{d\mathrm{T}}{dr}\right)\left(n - i\frac{\mathrm{T}}{r}\right)\frac{dw}{dr} + \frac{i}{r}\left[\frac{\mathrm{T}^2}{r^2} - \frac{d\mathrm{T}^2}{dr^2} - \left(n - i\frac{\mathrm{T}}{r}\right)^2\right]w\right\} \\ \text{where} \qquad \mathrm{D} &= \frac{2\mathrm{T}}{r}\left(\frac{\mathrm{T}}{r} + \frac{d\mathrm{T}}{dr}\right) - \left(n - i\frac{\mathrm{T}}{r}\right)^2 \end{aligned}\right\} (9).$$

For the particular case of $m = 0$, or motion in two dimensions (r, θ), it is convenient to put

$$\frac{-w}{m} = \phi \quad . \quad . \quad . \quad . \quad . \quad (10).$$

In this case the motion which superimposed on $\dot{r} = 0$ and $r\dot{\theta} = \mathrm{T}$ gives the disturbed motion is irrotational, and $\phi \sin(nt - i\theta)$ is its velocity-potential. It is also to be remarked that when m does not vanish the superimposed motion is irrotational where if at all, and only where, $\mathrm{T} = \text{const.}/r$, and that whenever it is irrotational ϕ as given by (10) is its velocity potential.

Eliminating ϱ and τ from (8) by (9) we have a linear differential equation of the second order for w. The integration of this, and substitutions of the result in (9), give w, ϱ, and τ, in terms of r and the two arbitrary constants of integration which, with m, n, and i, are to be determined to fulfil whatever surface conditions, or initial conditions, or conditions of maintenance, are prescribed for any particular problem.

Crowds of exceedingly interesting cases present themselves. Taking one of the simplest to begin :—

Case I.

Let $$\mathrm{T} = \omega r \ (\omega \text{ const.}) \quad . \quad . \quad . \quad (11),$$

$$\left.\begin{array}{l} \dot{r} = c \cos mz \sin (nt - i\theta), \text{ where } r = a \\ \dot{r} = \mathfrak{c} \cos mz \sin (nt - i\theta), \quad ,, \quad r = \mathfrak{a} \\ c, \mathfrak{c}, m, n, a, a' \text{ being any given quantities} \\ \quad \text{and } i \text{ any given integer} \end{array}\right\} \quad . \quad (12).$$

The condition $\mathrm{T} = \omega r$ simplifies (9) to

$$\left.\begin{array}{l} \mathfrak{r} = \dfrac{(n - i\omega)\left\{\left(n - i\omega\right)\dfrac{dw}{dr} - \dfrac{2i\omega}{r}w\right\}}{m\{4\omega^2 - (n - i\omega)^2\}} \\ \mathfrak{t} = \dfrac{(n - i\omega)\left\{2\omega\dfrac{dw}{dr} - \dfrac{i(n - i\omega)}{r}w\right\}}{m\{4\omega^2 - (n - i\omega)^2\}} \end{array}\right\} \quad . \quad (13),$$

and the elimination of $\mathfrak{r}$ and $\mathfrak{t}$ by these from (8) gives

$$-\frac{d^2w}{dr^2} + \frac{1}{r}\frac{dw}{dr} - \frac{i^2w}{r^2} + m^2\frac{4\omega^2 - (n - i\omega)^2}{(n - i\omega)^2}w = 0 \quad . \quad (14),$$

or

$$\left.\begin{array}{l} \dfrac{d^2w}{dr^2} + \dfrac{1}{r}\dfrac{dw}{dr} - \dfrac{i^2w}{r^2} + \nu^2w = 0 \\ \text{where} \quad \nu = m\sqrt{\dfrac{4\omega^2 - (n - i\omega)^2}{(n - i\omega)^2}} \end{array}\right\} \quad . \quad . \quad (15),$$

or

$$\left.\begin{array}{l} \dfrac{d^2w}{dr^2} + \dfrac{1}{r}\dfrac{dw}{dr} - \dfrac{i^2w}{r^2} - \sigma^2w = 0 \\ \text{where} \quad \sigma = m\sqrt{\dfrac{(n - i\omega)^2 - 4\omega^2}{(n - i\omega)^2}} \end{array}\right\} \quad . \quad . \quad (16).$$

Hence if J_i, $\mathfrak{J}_i$ denote Bessel's functions of order i, and of the first and second kinds,* that is to say J_i finite or zero for infinitely small values of r, and $\mathfrak{J}^i$ finite or zero for infinitely great values of r; and if I_i and $\mathfrak{I}_i$ denote the corresponding real functions with ν imaginary, we have

$$w = \mathrm{CJ}_i(\nu r) + \mathfrak{C}\mathfrak{J}_i(\nu r) \quad . \quad . \quad . \quad (17),$$

* Compare Proceedings, March 17, 1879, "Gravitational Oscillations of Rotating Water." Solution II. (Case of Circular Basons).

or $$w = \mathrm{CI}_i(\sigma r) + \mathfrak{C}\mathfrak{I}_i(\sigma r) \quad . \qquad (18);$$

where C and $\mathfrak{C}$ denote arbitrary constants, to be determined in the present case by the equations of condition (12). These are equivalent to $\varrho = c$ when $r = a$, and $\varrho = \mathfrak{c}$ when $r = \mathfrak{a}$, and, when (16) is used for w in (13), give two simple equations to determine C and $\mathfrak{C}$.

The problem thus solved is the finding of the periodic disturbance in the motion of rotating liquid in a space between two boundaries which are concentric circular cylindric when undisturbed, produced by infinitely small simple harmonic normal motion of these boundaries distributed over them according to the simple harmonic law in respect to the co-ordinates z, θ. The most interesting Sub-case is had by supposing the inner boundary evanescent ($\mathfrak{a} = 0$), and the liquid continuous and undisturbed throughout the space contained by the outer cylindric boundary of radius a. This, as is easily seen, makes $w = 0$ when $r = 0$, except for the case $i = 1$, and essentially, without exception, requires that $\mathfrak{c}$ be zero. Thus the solution for w becomes

$$w = \mathrm{CJ}_i(\nu r) \quad . \quad . \quad . \qquad (19)$$

or $$w = \mathrm{CI}\ (\sigma r) \quad . \quad . \quad . \qquad (20);$$

and the condition $\varrho = c$ when $r = a$ gives, by (13),

$$\mathrm{C} = \frac{\nu^2 m}{\nu \mathrm{J}'_i(\nu a) - \dfrac{2i\omega}{(n - i\omega)a}\mathrm{J}_i(\nu a)} \quad . \quad . \qquad (21).$$

or the corresponding I formula.

By summation after the manner of Fourier we find the solution for any arbitrary distribution of the generative disturbance over the cylindric surface (or over each of the two if we do not confine ourselves to the Sub-case), and for any arbitrary periodic function of the time. It is to be remarked that (6) represents an undulation travelling round the cylinder with linear velocity na/i at the surface, or angular velocity n/i throughout. To find the interior effect of a *standing* vibration produced at the surface we must add to the solution (6), or any sum of solutions of the same type, a solution, or a sum of solutions in all respects the same, except with $-n$ in place of n.

It is also to be remarked that great enough values of i make ν^2

negative, and therefore ν imaginary; and for such the solutions in terms of σ and the I_i, $\mathfrak{I}_i$ functions must be used.

Case II.—Hollow Irrotational Vortex in a fixed Cylindric Tube.

Conditions—

$$\left.\begin{array}{ll} & T=\dfrac{c}{r};\ \dot{r}=0 \text{ when } r=a; \\ \text{and} & P+p=0 \text{ for the disturbed orbit, } r=\mathfrak{a}+\int\dot{r}_{\mathfrak{a}}dt \end{array}\right\} \quad (22),$$

$\mathfrak{a}$ and a being the radii of the hollow cylindric interior, or free boundary, and of the external fixed boundary, and $\dot{r}_{\mathfrak{a}}$ the value of $\dot{r}$ where r is approximately equal to $\mathfrak{a}$. The condition $T=c/r$ simplifies (9) and (14) to

$$\varrho=-\frac{1}{m}\frac{dw}{dr}, \text{ and } \tau=\frac{iw}{mr} \quad . \quad . \quad (23);$$

$$\frac{d^2w}{dr^2}+\frac{1}{r}\frac{dw}{dr}-\frac{i^2w}{r^2}-m^2w \quad . \quad . \quad (24),$$

and by (7) we have $\varpi=\dfrac{1}{m}\left(n-\dfrac{ic}{r^2}\right)w$ . . (25).

Hence $\qquad w=CI_i(mr)+\mathfrak{C}\mathfrak{I}_i(mr) \qquad (26);$

and the equation of condition for the fixed boundary (radial velocity zero there) gives

$$CI'_i(ma)+\mathfrak{C}\mathfrak{I}'_i(ma)=0 \quad . \quad (27).$$

To find the other equation of condition we must first find an expression for the disturbance from circular figure of the free inner boundary. Let for a moment r, θ be the co-ordinates of one and the same particle of fluid. We shall have

$$\theta=\int\dot{\theta}dt; \text{ and } r=\int\dot{r}dt+r_0,$$

where r_0 denotes the radius of the "mean circle" of the particle's path.

Hence to a first approximation,

$$\theta = \frac{ct}{r^2} \quad . \quad . \quad . \quad . \qquad (28),$$

and therefore, by (6)

$$\dot{r} = \varrho \cos mz \sin \left(n - \frac{ic}{r^2}\right) t \,;$$

whence

$$r = r_0 - \frac{\varrho}{n - \dfrac{ic}{r^2}} \cos mz \cos (nt - i\theta) \quad . \quad . \qquad (29),$$

Hence the equation of the free boundary is

$$r = \mathfrak{a} - \frac{\varrho_{(r=\mathfrak{a})}}{n - i\omega} \cos mz \cos (nt - i\theta) \quad . \qquad (30)$$

where

$$\omega = \frac{c}{\mathfrak{a}^2} \quad . \quad . \quad . \quad . \quad . \qquad (31).$$

Hence at (r, θ, z) of this surface we have, from $\mathrm{P} = \int \frac{\mathrm{T}^2 dr}{r}$, of (6) above,

$$\mathrm{P} = \frac{\mathrm{T}^2}{r} (r - \mathfrak{a})$$

$$= - \frac{c^2}{\mathfrak{a}^3} \frac{\varrho(r = \mathfrak{a})}{n - i\omega} \cos mz \cos (nt - i\theta) \quad . \quad . \qquad (32).$$

Hence, and by (6), and (26), and (25), and (23), the condition $\mathrm{P} + p = 0$ at the free boundary gives

$$\frac{c^2}{\mathfrak{a}^3} [\mathrm{CI}_i'(m\mathfrak{a}) + \mathfrak{C}\mathfrak{I}_i'(m\mathfrak{a})] + \frac{(n - i\omega)^2}{m} [\mathrm{CI}_i(m\mathfrak{a}) + \mathfrak{C}\mathfrak{I}_i(m\mathfrak{a})] = 0 \qquad (33).$$

Eliminating $\mathrm{C}/\mathfrak{C}$ from this by (27) we get an equation to determine n, by which we find

$$n = \omega(i \pm \sqrt{\mathrm{N}}) \quad . \quad . \quad . \qquad (34),$$

where N is an essentially positive numeric.

II.—Sub-case.

A very interesting Sub-case is that of $a = \infty$, which, by (27), makes $\mathrm{C} = 0$; and therefore, by (33), gives

$$\mathrm{N} = m\mathfrak{a} \frac{-\mathfrak{I}'(m\mathfrak{a})}{\mathfrak{I}(m\mathfrak{a})} \quad . \qquad (35).$$

Whether in Case II. or Sub-case II. we see that the disturbance consists of an undulation travelling round the cylinder with angular velocity

$$\omega\left(1+\frac{\sqrt{N}}{i}\right), \text{ or } \omega\left(1-\frac{\sqrt{N}}{i}\right)$$

or of two such undulations superimposed on one another, travelling round the cylinder with angular velocities greater than and (algebraically) less than the angular velocity of the mass of the liquid at its free surfaces by equal differences. The propagation of the wave of greater velocity is in the same direction as that in which the liquid revolves; the propagation of the other is in the contrary direction when $N > i^2$ (as it certainly is in some cases).

If the free surface be started in motion with one or other of the two principal angular velocities (34), or linear velocities $\mathfrak{a}\omega\left(1 \pm \frac{\sqrt{N}}{i}\right)$, and the liquid be then left to itself, it will perform the simple harmonic undulatory movement represented by (6), (26), (23). But if the free surface be displaced to the corrugated form (30) and then left free either at rest or with any other distribution of normal velocity than either of those, the corrugation will, as it were, split into two sets of waves travelling with the two different velocities $\mathfrak{a}\omega\left(1 \pm \frac{\sqrt{N}}{i}\right)$.

The case $i=0$ is clearly exceptional, and can present no undulations travelling round the cylinder. It will be considered later.

The case $i=1$ is particularly important and interesting. To evaluate N for it remark that

$$\left.\begin{aligned} &I_1(mr) = I_0'(mr) \\ \text{and}\quad &\mathfrak{I}_1(mr) = \mathfrak{I}_0'(mr) \end{aligned}\right\} \quad . \quad . \quad . \quad (36).$$

Now the general solution of (24) is

$$\left.\begin{aligned} w = &\left(E + D\log\frac{1}{mr}\right)\left(1 + \frac{m^2r^2}{2^2} + \frac{m^4r^4}{2^2.4^2} + \&c.\right) \\ &+ D\left(\frac{m^2r^2}{2^2}S_1 + \frac{m^2r^2}{2^2.4^2}S_2 + \&c.\right) \end{aligned}\right\} \quad . \quad (36),$$

where E and D are constants Hence according to our notation

$$I_0(mr) = 1 + \frac{m^2r^2}{2^2} + \frac{m^4r^4}{2^2.4^2} + \&c. \quad . \quad . \quad (37),$$

the constant factor being taken so as to make $I_0(0) = 1$.

Stokes * investigated the relation between E and D to make $w = 0$ when $r = \infty$ and found it to be

$$\left.\begin{array}{l} E/D = \log 8 + \pi^{-\frac{1}{2}}\Gamma'\tfrac{1}{2} = +2{\cdot}079442 - 1{\cdot}963510 = {\cdot}11593 \\ \text{or, to 20 places,} \quad E/D = {\cdot}11593\ 15156\ 58412\ 44881 \end{array}\right\} (38).$$

Hence, and by convenient assumption for constant factor,

$$\left.\begin{array}{l} \mathfrak{I}_0(mr) = \log \dfrac{1}{mr}\left(1 + \dfrac{m^2r^2}{2^2} + \dfrac{m^4r^4}{2^2.4^2} + \&c.\right) \\ \qquad + \dfrac{m^2r^2}{2^2}(S_1 + {\cdot}11593) + \dfrac{m^4r^4}{2^2.4^2}(S_2 + {\cdot}11593) + \&c. \end{array}\right\} (39).$$

It is to be remarked that the series in (36) and (39) are convergent however great be mr; though for values of mr exceeding 6 or 7 the semi-convergent expressions † will give the values of the functions nearly enough for most practical purposes, with much less arithmetical labour.

From (37) and (39) we find by differentiation

$$\left.\begin{array}{l} I_1(mr) = \dfrac{mr}{2} + \dfrac{m^3r^3}{2^2.4} + \dfrac{m^5r^5}{2^2.4^2.6} + \&c. \\ I_1'(mr) = \dfrac{1}{2} + \dfrac{3m^2r^2}{2^2.4} + \dfrac{5m^4r^4}{2^2.4^2.6} + \&c. \end{array}\right\} \quad . \quad . \quad . \quad (40).$$

* "On the Effect of Internal Friction on the Motion of Pendulums," equations (93) and (106).—*Cambridge Phil. Trans., Dec.* 1850.

P.S.—I am informed by Mr J. W. L. Glaisher that Gauss, in section 32 of his "Disquisitiones Generales circa seriem infinitam $1 + \frac{\alpha.\beta}{1.\gamma}x$ + &c.," (Opera, vol. iii. p. 155), gives the value of $-\pi^{-\frac{1}{2}}\Gamma'\frac{1}{2}$, or $-\psi(-\frac{1}{2})$ in his notation, to 23 places as follows:—

$$1{\cdot}96351\ 00260\ 21423\ 47944\ 099.$$

Thus it appears that the last figure in Stokes' result (106) ought, as in the text, to be 0 instead of 2. In Callet's Tables we find

$$\log_e 8 = 2{\cdot}07944\ 15416\ 79835\ 92825,$$

and subtracting the former number from this we have the value of E to 20 places given in the text.

† Stokes, *ibid.*

$$\left.\begin{aligned}\mathfrak{K}_1(mr) &= \frac{1}{mr} - \frac{mr}{2^2}[-1+2(S_1+\cdot1159315)] + \frac{m^3r^3}{2^2.4^2}[-1+2(S_2+\cdot1159315)] + \&c. \\ &\quad - \log\frac{1}{mr}\left(\frac{mr}{2} + \frac{m^3r^3}{2^2.4} + \frac{m^5r^5}{2^2.4^2.6} + \&c.\right) \\ \mathfrak{K}_1'(mr) &= \frac{-1}{m^2r^2} - \frac{1}{2^2}[-3+2(S_1+\cdot1159315)] + \frac{m^2r^2}{2^2.4^2}[7-6(S_2-\cdot1159315)] + \&c. \\ &\quad - \log\frac{1}{mr}\left(\frac{1}{2} + \frac{3m^2r^2}{2^2.4} + \frac{5m^4r^4}{2^2.4^2.6} + \&c.\right)\end{aligned}\right\} (41).$$

For an illustration of Case II. with $i=1$, suppose $m\mathfrak{a}$ to be very small. Remarking that $S_1=1$, we have

$$N = \frac{-m\mathfrak{a}\mathfrak{K}_1'(m\mathfrak{a})}{\mathfrak{K}_1(m\mathfrak{a})} = \frac{1+\dfrac{m^2\mathfrak{a}^2}{2}\left[\log\dfrac{1}{m\mathfrak{a}} - \frac{1}{2} + \cdot1159\right]}{1-\dfrac{m^2\mathfrak{a}^2}{2}\left[\log\dfrac{1}{m\mathfrak{a}} + \frac{1}{2} + \cdot1159\right]}$$

$$= 1 + m^2\mathfrak{a}^2\left(\log\frac{1}{m\mathfrak{a}} + \cdot1159\right) \quad . \quad (42);$$

Hence in this case at all events $N > i^2$; and the angular velocity of the slow wave, in the reverse direction to that of the liquid's revolution, is

$$-n = \tfrac{1}{2}\omega m^2\mathfrak{a}^2\left(\log\frac{1}{m\mathfrak{a}} + \cdot1159\right) \quad . \quad . \quad (43).$$

This is very small in comparison with

$$2\omega + \tfrac{1}{2}\omega m^2\mathfrak{a}^2\left(\log\frac{1}{m\mathfrak{a}} + \cdot1159\right) \quad . \quad . \quad . \quad (44),$$

the angular velocity of the direct wave; and therefore clearly if the initial normal velocity of the surface when left free after being displaced from its cylindrical figure of equilibrium be zero or anything small, the amplitude of the quicker direct wave will be very small in proportion to that of the reverse slow one.

Case III.

A slightly disturbed vortex column in liquid extending through all space between two parallel planes; the undisturbed column consisting of a core of uniform vorticity (that is to say, rotating like a solid) surrounded by irrotationally revolving liquid with no slip at

the cylindric interface. Denoting by a the radius of this cylinder we have

$$\left.\begin{aligned} &\mathrm{T} = \omega r\ , \text{ where } r<a \\ \text{and}\qquad &\mathrm{T} = \omega\frac{a^2}{r}\quad ,, \quad r>a \end{aligned}\right\} \quad . \quad . \quad (45).$$

Hence (13), (14) hold for $r<a$, and (23), (24) for $r>a$.

Going back to the form of assumption (6) we see that it suits the condition of rigid boundary planes if Oz be perpendicular to them, O in one of them, and the distance between them π/m.

The conditions to be fulfilled at the interface between core and surrounding liquid are that ϱ and w must have the same values on the two sides of it: it is easily proved that this implies also equal values of τ on the two sides. The equality of ϱ on the two sides of the interface gives, by (13) and (23),

$$\left\{\frac{\left(i\omega - n\right)\left[\left(i\omega - n\right)\dfrac{dw}{dr} + \dfrac{2i\omega}{r}w\right]}{4\omega^2 - (i\omega - n)^2}\right\}^{\text{internal}}_{r=a} = -\left(\frac{dw}{dr}\right)^{\text{external}}_{r=a} \quad . \quad (46):$$

and from this and the equality of w on the two sides we have

$$\frac{\left(i\omega - n\right)\left[\left(i\omega - n\right)\left(\dfrac{dw}{wdr}\right)^{\text{internal}}_{r=a} + \dfrac{2i\omega}{a}\right]}{4\omega^2 - (i\omega - n)^2} = -\left(\frac{dw}{wdr}\right)^{\text{external}}_{r=a} \quad . \quad (47).$$

The condition that the liquid extends to infinity all round makes $w=0$ when $r=\infty$. Hence the proper integral of (24) is of the form $\mathfrak{I}_i$: and the condition of undisturbed continuity through the axis shows that the proper integral of (13) is of the form J_i. Hence

$$\left.\begin{aligned} &w = \mathrm{CJ}_i\,(\nu r) \text{ for } r<a \\ \text{and}\qquad &w = \mathfrak{C}\mathfrak{I}_i\,(mr)\ ,, \quad r>a \end{aligned}\right\} \quad . \quad . \quad (48);$$

by which (47) becomes

$$\frac{\left(i\omega - n\right)\left[\left(i\omega - n\right)\dfrac{\nu \mathrm{J}_i'(\nu a)}{\mathrm{J}_i(\nu a)} + \dfrac{2i\omega}{a}\right]}{4\omega^2 - (i\omega - n)^2} = \frac{-m\mathfrak{I}_i'(ma)}{\mathfrak{I}_i(ma)} \quad (49);$$

or by (15),

$$\frac{\mathrm{J}_i'(q)}{q\mathrm{J}_i(q)} + \frac{i}{q^2\lambda} = \frac{-\mathfrak{I}_i'(ma)}{ma\mathfrak{I}_i(ma)} \quad . \quad . \quad (50),$$

where $$\lambda = \frac{i\omega - n}{2\omega} \quad . \quad . \quad . \quad . \quad (51),$$

and $$q^2 = m^2a^2\frac{1-\lambda^2}{\lambda^2} \quad . \quad . \quad . \quad (52).$$

Remarking that $J_i(q)$ is the same for positive and negative values of q, and that it passes from positive through zero to a finite negative maximum, thence through zero to a finite positive maximum and so on an infinite number of times, while q is increased from 0 to ∞, we see that while λ is increased from -1 to 0 the first member of (50) passes an infinite number of times continuously through all real values from $-\infty$ to $+\infty$: and that it does the same when λ is diminished from $+1$ to 0. Hence (50), regarded as a transcendental equation in λ, has an infinite number of roots between -1 and 0 and an infinite number between 0 and $+1$. And it has no roots except between -1 and $+1$, because its second member is clearly positive, whatever be ma; and its first member is essentially real and negative for all real values of λ except between -1 and $+1$, as we see by remarking that when $\lambda^2 > 1, -q^2$ is real and positive, and $-J'_i(q)/qJ_i(q)$ is real and $> i/(-q^2)$, while $i/q^2\lambda$, whether positive or negative, is of less absolute value than $i/(-q^2)$.

Each of the infinite number of values of λ yielded by (50) gives, by (51) and (13), a solution of the problem of finding simple harmonic vibrations of a columnar vortex, with m of any assumed value. All possible simple harmonic vibrations are thus found: and summation after the manner of Fourier for different values of m, with different amplitudes and epochs and different epochs, gives every possible motion, deviating infinitely little from the undisturbed motion in circular orbits.

The simplest Sub-case, that of $i = 0$, is curiously interesting. For it (50), (51), (52) give

$$\frac{J_0'(q)}{qJ_0(q)} = \frac{-\mathfrak{I}_0'(ma)}{ma\mathfrak{I}_0(ma)} \quad . \quad . \quad . \quad (53),$$

and $$n = \frac{2\omega ma}{\sqrt{(m^2a^2+q^2)}} \quad . \quad . \quad . \quad (54).$$

The successive roots of (53), regarded as a transcendental equation in q, lie between the 1st, 3d, 5th - - - roots of $J_0(q) = 0$, in order of ascending values of q, and the next greater roots of $J'_0(q) = 0$,

coming nearer and nearer down to the roots of J_0, the greater they are. They are easily calculated by aid of Hansen's Tables of Bessel's functions J_0 and J_1 (which is equal to J'_0) from $q=0$ to $q=20$.* When ma is a small fraction of unity, the second member of (53) is a large number, and even the smallest root exceeds by but a small fraction the first root of $J_0(q)=0$, which, according to Hansen's Table, is 2·4049, or approximately enough for the present 2·4. In every case in which q is very large in comparison with ma, whether ma is small or not, (54) gives

$$n=\frac{2\omega ma}{q} \text{ approximately.}$$

Now going back to (6) we see that the summation of two solutions to constitute waves propagated along the length of the column, gives

$$\left.\begin{array}{ll} \dot{r}=-\varrho\sin(nt-mz)\,; & r\dot{\theta}=\mathrm{T}+\tau\cos(nt-mz) \\ \dot{z}=w\cos(nt-mz)\,; & p=\mathrm{P}+\varpi\cos(nt-mz) \end{array}\right\} \quad (55).$$

The velocity of propagation of these waves is n/m. Hence when q is large in comparison with ma, the velocity of longitudinal waves is $2\omega a/q$, or $2/q$ of the translational velocity of the surface of the core in its circular orbit. This is 1/1·2, or $\frac{5}{6}$ of the translational velocity, in the case of ma small, and the *mode* corresponding to the smallest root of (53). A full examination of the internal motion of the core, as expressed by (55), (13), (48), (15) is most interesting and instructive. It must form a more developed communication to the Royal Society.

The Sub-case of $i=1$, and ma very small, is particularly interesting and important. In it we have, by (42), for the second member of (50), approximately,

$$\frac{-\mathfrak{I}_1'(ma)}{ma\mathfrak{I}_1(ma)}=\frac{1}{m^2a^2}\left[1+m^2a^2\left(\log\frac{1}{ma}+\cdot1159\right]\right) . \quad (56).$$

In this case the smallest root, q, is comparable with ma, and all the others are large in comparison with ma. To find the smallest, remark that, when q is very small, we have to a second approximation,

$$\frac{J_1'(q)}{qJ_1(q)}=\frac{1}{q^2}-\frac{1}{4} \quad . \quad . \quad . \quad (57).$$

* Republished in Lömmel's "Besselsche Functionen," Leipzig, 1868.

Hence (50), with $i=1$, becomes, to a first approximation,

$$\frac{1}{q^2}\left(1+\frac{1}{\lambda}\right)=\frac{1}{m^2a^2} \quad . \quad . \quad . \quad (58).$$

This and (52) used to find the two unknowns λ and q^2, give

$$\lambda=\tfrac{1}{2}, \text{ and } q^2=3m^2a^2,$$

for a first approximation. Now, with $i=1$, (51) becomes

$$\lambda=\tfrac{1}{2}\left(1-\frac{n}{\omega}\right),$$

and therefore n/ω is infinitely small. Hence (52) gives for a second approximation

$$q^2=3m^2a^2\left(1+\frac{8n}{3\omega}\right) \quad . \quad . \quad . \quad (59);$$

and we have

$$\frac{1}{q^2\lambda}=\frac{2}{3}\frac{1}{m^2a^2}\left(1-\frac{5n}{3\omega}\right) \quad . \quad . \quad . \quad (60).$$

Using now (57), (59), (60), and (56) in (50), we find to a second approximation'

$$\frac{1}{3ma^2}\left(1-\frac{8n}{3\omega}\right)-\frac{1}{4}+\frac{2}{3ma^2}\left(1-\frac{5n}{3\omega}\right)=\frac{1}{m^2a^2}\left[1+m^2a^2\left(\log\frac{1}{ma}+\cdot 1159\right)\right];$$

whence

$$\frac{-n}{\omega}=\frac{1}{2}m^2a^2\left(\log\frac{1}{ma}+\frac{1}{4}+\cdot 1159\right) \quad . \quad . \quad (61).$$

Compare this result with (43) above. The fact that, as in (43), $-n$ is positive in (61), shows that in this case also the direction in which the disturbance travels round the cylinder is *retrograde* (or opposite to that of the translation of fluid in the undisturbed vortex); and, as is to be expected, the values of $-n$ are approximately equal in the two cases, when ma is small enough; but it is smaller by a relatively small difference in (60) than in (43), as is also to be expected.

The case of ma small and $i>1$ has a particularly simple approximate solution for the smallest q-root of the transcendental (50). With any value of i instead of unity we still have (58), as a first approximation for q small. Eliminating q^2/m^2a^2 between this and (52) we still find $\lambda=\frac{1}{2}$; but instead of $n=0$ by (51), we now have $n=(i-1)\omega$. Thus is proved the solution for waves of deformation of sectional figure travelling round a cylindrical vortex, announced thirteen years ago without proof in my first article respecting Vortex Motion.*

* "Vortex Atoms," Proc. Roy. Soc. Edin., Feb. 18, 1867.

4. The Structure of the Comb-like Branchial Appendages and the Teeth of the Basking Shark (*Selache maxima*). By Professor Turner, M.B., F.R.S.

Attention was drawn to the statements made on the position of peculiar comb-like fringes on the branchiæ of the basking shark by Gunnerus, Pennant, Low, Mitchell, Foulis, Brito Capello, Cornish, Steenstrup, Pavesi, P. & H. Gervais, Percival Wright, and Allman, and to the structure of these fringes by Hannover and MM. Gervais. The author then proceeded to give a detailed description of the structure of the plates forming these fringes from a specimen presented to him by the Rev. M. Harvey of St. John's, Newfoundland, the general summary of which is as follows: the whole periphery of a plate consisted of a hard unvascular dentine, the tubes in which were very distinctive; in a considerable part of the shaft these tubes arose from a single central pulp cavity, but in the semilunar attached base of the plate the single central cavity did not exist, but was replaced by a set of anastomosing vascular canals, which collectively represented a pulp cavity, and which gave origin to numerous characteristic dentine tubes. It was suggested that these plates were developed in the mucous membrane covering the branchiæ after the manner of teeth. Although these plates act, like whalebone plates, to separate from the water the small organisms on which this shark lives, they were shown to be essentially different in structure and mode of origin, the matrix of whalebone being a cornification of the ephithelium of the palate derived from the epiblast, whilst the matrix in the shark's branchial plates is a calcification of dermal or sub-epithelial structure, and therefore derived from the mesoblast. Reference was made to the observations of Andrew Smith on *Rhinodon*, in which an apparatus having a similar office, but probably a different structure, was seen in that shark; and to the observations of Van Beneden on a comb-like fringe found fossilised in the Antwerp Crag.

The structure of the small conical teeth of the basking shark was then described from a specimen also presented by the Rev. M. Harvey. They were shown to have an external layer of hard unvascular dentine, covering an extensive core in which relatively large

vascular canals anastomosed to form a network. From these canals numerous dentine tubes arose. Both the core of the tooth and the anastomosing canals with their dentine tubes in the semi-lunar base of a comb-plate were examples of vaso-dentine. These plates in the basking shark may be regarded as an example of excessively developed branchial teeth, a development which is co-related with the small size and simple form of the maxillary and mandibular teeth, with the non-predaceous habits of the fish, and with the particular nature of the food on which it lives.

This communication will be printed *in extenso* in the "Journal of Anatomy and Physiology," April 1880.

5. Preliminary Report on the Tunicata of the "Challenger" Expedition. By W. A. Herdman, B.Sc., Baxter Scholar in Natural Science in the University of Edinburgh.

(*By permission of the Lords Commissioners of the Treasury.*)

I. Ascidiadæ.

Last year the Tunicata collected during the "Challenger" Expedition were entrusted to me for description by Sir Wyville Thomson.

The entire collection comprises from 150 to 200 species, the majority of which are new to science.

As yet only some of the more abnormal *Synascidiæ* and about half the *Ascidiæ simplices* have been carefully examined. The present paper is the preliminary report on the Ascidiadæ, the first family of the *Ascidiæ simplices.*

The family Ascidiadæ is synonymous with Savigny's genus *Phallusia*, or Forbes' *Ascidia*, and includes those simple ascidians which are, as a rule, externally characterised by an eight-lobed branchial and a six-lobed atrial aperture, as distinguished from two other families—the Cynthiadæ, with both apertures four-lobed, and the Molgulidæ, having the branchial six- and the atrial four-lobed.

This point, the number of lobes round the apertures, though a most important diagnostic, does not hold good for all Ascidiadæ without exception. Indeed, any one of the characters of the three families, if employed singly, will be found, while sufficing in the

majority of cases, to break down in regard to a few species. For example, *Ascidia involuta* has the entire body encrusted with sand grains and shells, a condition characteristic of the MOLGULIDÆ; simple unbranched tentacles, an important character in the ASCIDIADÆ are also found in *Styela* (CYNTHIADÆ); lastly, the papillated branchial sac of the ASCIDIADÆ can no longer be considered an essential, *Abyssascidia* n. gen., having no papillæ on its longitudinal bars.

The more important characteristics of the ASCIDIADÆ are the following:—

Body sessile, attached.

Branchial aperture eight-lobed, *atrial* six-lobed.

Test gelatinous or cartilaginous.

Branchial sac not conspicuously folded; papillated.

Tentacles unbranched, filiform.

The family includes five known genera, two of which—*Rhopalæa* and *Rhodosoma*—are not represented in the "Challenger" collection, which however includes a new genus—*Abyssascidia*, and a new sub-genus of *Ascidia*—*Pachychlæna*.

The "Challenger" ASCIDIADÆ are divided into the following genera:—

(1.) *Ciona*, Fleming, 1 species.

(2.) *Ascidia.*, Linn., 8 species.

Pachychlæna, n. sub-gen., 3 species.

(3.) *Abyssascidia*, n. gen., 1 species.

(4.) *Corella*, Hancock, 1 species.

Of these fourteen species, twelve are new to science. The majority of the specimens are from shallow water (10 to 100 fathoms); two (*Ascidia tenera* and *Ascidia meridionalis*) are from moderate depths (245 and 600 fathoms); while one (*Abyssascidia wyvillii*) was obtained at the great depth of 2600 fathoms.

The following table* shows the different genera and species synoptically, and gives a few of their more important distinctive characters.

* On account of the meagre, and in some cases insufficient, manner in which ascidians have often been described, it has been found impossible to extend the table so as to include the species already known.

ASCIDIADÆ.

- Branchial aperture eight-lobed.
  - Viscera on right side. Stigmata curved.
    - *Corella.*
      - *C. japonica.*
  - Viscera on left side. Stigmata straight.
    - Viscera extending beyond branchial sac posteriorly. Dorsal lamina in the form of languets.
      - *Ciona.*
        - *C. flemingi.*
    - Viscera of same length as branchial sac. Dorsal lamina in the form of a membrane.
      - *Ascidia.*
        - Test normal.
          - Tentacles crowded, not long and short alternately.
            - Branchial sac plicated. Mantle forming a crest between tubes.
              - *A. pyriformis.*
            - Branchial sac not plicated. No crest on mantle.
              - *A. falcigera.*
          - Tentacles long and short alternately.
            - Branchial sac not plicated.
              - *A. tenera.*
            - Branchial sac plicated.
              - Small intermediate papillæ; olfactory tubercle normal.
                - Test light-coloured.
                  - Tentacles 60.
                    - *A. meridionalis.*
                  - Tentacles 30.
                    - *A. mentula.*
                - Test indigo-coloured.
                  - *A. nigra.*
              - No intermediate papillæ; olfactory tubercle serpentiform.
                - *A. translucida.*
        - Test very thick and solid.
          - *Pachychlæna* (sub-gen.).
            - Test dark-coloured. Tentacles long and short alternately. Dorsal lamina pectinated.
              - Tentacles, 30–35.
                - *P. obesa.*
              - Tentacles, 60 long and 60 very short.
                - *P. oblonga.*
            - Test light-coloured. Tentacles not long and short alternately. Dorsal lamina not pectinated.
              - *P. gigantea.*
- Branchial aperture more than eight-lobed.
  - *Abyssascidia.*
    - *A. wyvillii.*

Ciona flemingi, n. sp.

External appearance.—Shape somewhat pyriform, elongated; anterior end wide, posterior much narrower, forming a short stalk turned ventrally and attached to a fragment of nullipore by the extremity of its right side. Apertures at the anterior end, inconspicuous; the branchial near the ventral edge, the atrial near the dorsal edge. They are equally far forward, the most anterior point being placed between them. Surface smooth. Colour light-grey. Length, 2·2 cm.; breadth, 8 mm.

Test thin, soft, almost gelatinous, transparent; vessels few.

Mantle normal; musculature rather feebly developed, consisting chiefly of a few straight bundles running longitudinally.

Branchial sac rather thick, small and shrunken looking; internal longitudinal vessels coarse and strong, much crumpled, bearing knob-like papillæ at their intersections with the transverse vessels; no intermediate papillæ; stigmata elongate-elliptical, two or three in a mesh.

Dorsal lamina reduced to a series of short tusk-like languets.

Tentacles simple, all one length, twelve in number.

Olfactory tubercle heart-shaped.

Viscera extending beyond the branchial sac posteriorly.

A single specimen labelled "Off Gomera, 75 fms."

(*Pachychlæna*, n. sub-gen. of *Ascidia.*)

Pachychlæna oblonga, n. sp.

External appearance.—Shape irregularly oblong, widest about the middle, narrowing somewhat towards the anterior end, which is obtuse and flattened; posterior end rather drawn out, attached to the interior of a large *Cardium*, which is in a three-quarters closed condition, constricting the test of the ascidian. Branchial aperture not terminal, placed on the right side near the ventral edge and about one-fifth of the distance to the mouth of the shell; it is directed ventrally posteriorly and to the right. Atrial aperture on the right side, near the dorsal margin and slightly anterior to the branchial aperture; it is directed dorsally and anteriorly. Seen from the ventral aspect it seems as if the anterior end had been bent over towards the right side, thus accounting for the lateral position of the branchial aperture.

Surface smooth but mamillated, very strongly on the anterior half, and especially near the branchial aperture where a few more sharply cut papillæ are visible. Colour light smoky brown, rather deeper in tint at the anterior end. Length, 8 cm; breadth, 4 cm.

Test cartilaginous, thickish, of a light greyish-brown colour throughout. Vascular trunks enter the test on the left side about halfway down, and large vessels ramify on the inner surface.

Mantle moderately muscular.

Branchial sac plicated longitudinally; the transverse vessels divide the grooves into rows of pouches, which are rather irregularly placed, and have no relation to the internal longitudinal bars. Transverse vessels all nearly of one size; meshes transversely oblong, containing each eight to ten stigmata; papillæ large and irregularly shaped, no intermediate smaller ones.

Dorsal lamina ribbed transversely, and strongly pectinated at the margin, a rib running out to the apex of each tooth.

Tentacles numerous, filiform, sixty-two large ones and about the same number of very minute intermediate ones. These last are so small as to be easily overlooked; usually one is placed between each pair of large tentacles, but in some spaces there appears to be none.

Olfactory tubercle large, irregularly oval in outline.

One specimen, in excellent condition, from Station 162 (Bass' Straits), 38 to 40 fathoms.

This species agrees in the thickness of its test with the next two species, and in the structure of the branchial sac with these and some others. The difference of size in the tentacles, the condition of the dorsal lamina, the transversely oblong meshes, and the absence of intermediate papillæ are all important characters.

Pachychlœna obesa, n. sp.

External appearance.—Shape unknown, on account of the absence of the greater part of the test, probably oval or irregularly spherical. Apertures not distant, depressed. Surface smooth, mamillated. Colour dark earthy-brown. Length probably about 10 cm.; breadth about 6 cm.

Test cartilaginous, thick (8 mm.), solid, opaque; vessels visible on the internal surface.

Mantle thick on the right (branchial) side of the body, and on the tubes, but not very muscular; membranous on the large distended left (visceral) side. Tubes long and rather narrow.

Branchial sac long and narrow, pointed at the posterior end; plicated longitudinally, the internal longitudinal bars being placed on the ridges. Meshes transversely oblong, each containing about six stigmata. Papillæ all of one size, irregular in shape, often cleft or lobed.

Dorsal lamina ribbed transversely, and bearing small teeth on its free margin.

Tentacles filiform, slender, alternately large and small; probably 30 to 35 in number.

Two specimens from Station 162 (Bass' Straits), 38 to 40 fathoms. In both specimens almost the whole of the test is gone, which makes it impossible to give the external characters with certainty. The branchial aperture also is damaged in both to such an extent that the exact number of tentacles and the nature of the olfactory tubercle cannot be determined. This species is closely allied to the last.

Pachychlæna gigantea, n. sp.

External appearance.—Shape, as far as can be made out, irregularly oblong, the right side being larger than the left. Probably attached by the posterior part of the ventral edge. Branchial aperture terminal, on a large irregularly rounded projection turned towards the left side. Atrial aperture on the dorsal edge, also on a large projection, situated more than one-third of the way down. Lobes of apertures irregular but prominent. Surface very irregular and almost covered by *Polyzoa*, *Hydroida*, *Algæ*, &c. Colour of a warm yellowish-grey where the test itself is seen. Length about 12 cm.; breadth, 5 to 7 cm.

Test cartilaginous, very thick (2 mm. to 4 cm.) and solid, white in mass with a hyaline tint where thin, yellowish-grey on the external surface. Large vessels ramify in the inner layer; the vascular trunks probably enter the test at the base of the right side towards the ventral edge.

Mantle strongly muscular over the right side and on the tubes,

membranous on the left side which is large and projecting. Tubes very long and diverging at more than a right angle.

Branchial sac very thick, coarse, and opaque, of a brown colour. Plicated longitudinally, and the grooves divided into pouches as in the last two species. On the external aspect the wide transverse vessels are connected by equally wide irregularly placed longitudinal vessels, thus forming a network of quadrangular meshes, each of which contains about four rows of stigmata. Meshes on the internal surface much elongated transversely, each containing 15 to 20 stigmata. Papillæ at the corners, no smaller intermediate ones.

Dorsal lamina wide, strongly ribbed transversely, but not pectinated.

Tentacles long and stout, about 60 in number, large and small not alternating.

Olfactory tubercle heart-shaped, $3\frac{1}{2}$ mm. long.

Two specimens from Simon's Bay, 10 to 20 fathoms.

In both, half the test has been cut away; the ventral edge, the posterior end, and part of each side is wanting.

This species and the two preceding are allied forms. They agree in the great thickness and solidity of the test, in the transversely elongated meshes of the branchial sac, and in the absence of small intermediate papillæ. They also possess that minute longitudinal plication of the stigmatic part of the branchial sac which is found in two other new species (*Ascidia pyriformis* and *Ascidia translucida*), and on account of which Verrill proposed to separate *Ascidia complanata* under the generic title of *Ascidiopsis*. This structure, however, is also to be seen in *Ascidia mentula*, *Ascidia sordida*, and several other species, some of which differ from each other in important points. On account of this, I think it unadvisable to use the plication of the branchial sac as a characteristic in breaking up *Ascidia*. If any division of the genus is necessary, the three species just described form a very natural section characterised by the several points of resemblance mentioned above, and worthy of being separated, not on account of the similarity in structure of their branchial sacs, but because of the remarkable thickness and solidity of their tests suggesting *Pachychlœna* (παχύς and χλαῖνα) as an appropriate subgeneric name.

Ascidia meridionalis, n. sp.

External appearance.—Shape somewhat variable, generally oval, the anterior end being slightly narrower than the posterior, flattened laterally, base rounded; attached by posterior end and part of left side. Branchial aperture terminal, placed on a large conical papilla of which the apex is inclined ventrally and to the right. Atrial aperture to the right of or on the dorsal edge, and about one-third of the way down, slightly projecting. Surface slightly velvety, with minute processes scattered over it. Colour light brown or horn-coloured. Length about 12 cm.; breadth about 8 cm.

Test softish, tears easily, from 1·5 to 6 mm. thick, the left side being thicker than the right. Vascular trunks enter about the middle of the left side near the ventral margin, large vessels visible on the inner surface, which is smooth and shining.

Mantle moderately muscular.

Branchial sac minutely undulated longitudinally. Three small transverse vessels between each pair of large ones. Papillæ, and generally smaller intermediate ones present.

Dorsal lamina broad, ribbed transversely.

Tentacles simple, filiform, about 60 in number, placed long and short alternately.

Olfactory tubercle semilunar, horns pointing anteriorly.

Several specimens from Station 320 (off the coast of Buenos Ayres), 600 fathoms, and two specimens from Station 313 (Strait of Magellan), 55 fathoms.

Ascidia mentula, O. F. Müller.

This species was obtained at four localities at Kerguelen Island, in depths of from 10 to 60 fathoms.

Ascidia vasculosa, n. sp.

External appearance.—Shape very irregular, somewhat quadrangular, depressed; anterior end a little prolonged and narrowed. Attached by the left side near the base. Branchial aperture not quite terminal, being on the right side of the anterior extremity. Atrial aperture also on the right side, nearer the dorsal than the

ventral edge, and a little in front of the middle. Both apertures rather depressed and concealed. Surface very irregular, grooved and mamillated; *Synascidiæ*, annelide-tubes, &c., adhering to it. Colour light yellowish-grey, not opaque, rather hyaline at the edges, and showing everywhere numerous blood-vessels ramifying near the surface. The terminal twigs of the vessels with their swollen ends are a prominent feature. Length, 9 cm.; breadth, 5·6 cm.

Test solid looking, varies in thickness from less than ·5 mm. on the right side behind the middle to 1·5 cm. on the left side near the place of attachment. Apertures lobed indistinctly; vascular trunks enter on the left side near the ventral edge and branch usually dichotomously, the terminal twigs ending in swollen knobs. The test shows no bladder cells. It contains the small spherical fusiform and stellate cells, and many minute granules. Crystals or concretions are also present, generally in the form of short rods and crosses.

One specimen, the test only, from Royal Sound, Kerguelen Island, 28 fathoms.

It may be considered a somewhat doubtful proceeding to describe an ascidian from the test alone, and certainly in most cases it would not be proper. Still this specimen possessed such well-marked characteristics that I was tempted to give it a name. I believe that the test is distinct from that of all known species, and that when other specimens are found they will be easily recognised. It differs from *Ascidia arachnoidea* in the general shape and the position of the apertures.

Ascidia nigra, Savigny.

One specimen labelled, "Bermuda, shallow water." Length, 6 cm.; breadth, 4 cm.

Ascidia translucida, n. sp.

External appearance.—Shape ellipsoidal, oblong-ovate to oblong; both ends rounded. Attached slightly by left side near base. Apertures sessile, placed on the right side. Branchial nearly terminal and median. Atrial more than a third of the way down, midway between the centre and the dorsal edge. Surface smooth and glossy. Colour very light grey, almost transparent, marked on the left side and the margins by white vascular ramifications. Length, 2·2 cm.; breadth, 1·2 cm.

Test moderately thick and solid, transparent. Vascular trunks enter near the centre of the left side, are of large size and branch freely; none visible on the centre of the right side.

Mantle thin.

Branchial sac longitudinally plicated, showing externally a division into pouches. Meshes nearly square, the transverse extent being slightly the longer; each contains six to eight stigmata. In the centre of each mesh a vertical oblique vessel connects the inter-papillar membrane at the top of the mesh with the transverse vessel at the bottom. Papillæ at the corners long and conical, no intermediate ones.

Dorsal lamina ribbed transversely, edge plain.

Tentacles simple, 30 to 35, long and short alternately.

Olfactory tubercle greatly elongated laterally, disposed in a series of irregular folds.

Three specimens from Kerguelen Island, 28 fathoms. In one specimen the vascular ramifications in the test are more conspicuous than in the other two.

Ascidia tenera, n. sp.

External appearance.—Shape oblong, flattened laterally; posterior end rounded, anterior end rather blunt; attached by the posterior third of the left side. Branchial aperture terminal, directed somewhat ventrally, sessile, lobes well marked. Atrial aperture placed to the right of the dorsal border, about one-third of the way down, sessile, lobes well marked. Surface soft and somewhat velvety, marked with slight creases, mostly longitudinal; near the apertures, especially the branchial, raised into minute pointed projections. Colour light brownish grey or pale horn-colour. Length, 5 cm.; breadth, 3 cm.

Test thin, soft, easily torn, transparent. Vessels moderately developed, trunks enter on the left side near the base.

Mantle very thin, muscular bands delicate, course of alimentary canal visible from both sides.

Branchial sac not plicated. Generally five or seven smaller transverse vessels between a pair of larger ones. Longitudinal internal bars narrow but well marked, bearing papillæ at the

corners of the meshes, and smaller more conical intermediate ones. Stigmata elongated, three or four in each mesh.

Dorsal lamina rather broad, delicately ribbed transversely; edge pectinated, having a small intermediate tooth between each pair of larger ones.

Tentacles filiform, 40 in number, long and short alternately.

Three specimens: one in good condition from Station 311 (West end of Strait of Magellan), 245 fathoms; and two, one of them damaged, from Station 320 (off the coast of Buenos Ayres), 600 fathoms. The two latter are slightly smaller than the dimensions given above.

This species somewhat resembles *Ascidia virginea*, but is undoubtedly distinct from it. That species differs from the present one chiefly in its greater length, its (slightly) greater number of tentacles, the absence of intermediate papillæ, and in the condition of the dorsal lamina—all good characters.

Ascidia pyriformis, n. sp.

External appearance.—Irregularly pear-shaped, anterior end narrow, posterior broad and rounded. Attached by a small area near the posterior end of the left side. Branchial aperture terminal, placed on a long somewhat conical projection turned dorsally. Sides of this projection channelled by eight grooves leading down from between the lobes of the aperture. A strong elevated ridge extending from the base of the projection along the anterior part of the dorsal edge. Atrial aperture sessile, placed at the posterior extremity of this ridge, being more than half-way from the anterior to the posterior end. Surface irregular, prolonged into a few thickish processes for attachment at the base, slightly rough, the globular posterior end encrusted with sand and shell fragments. Colour dull dirty grey. Length, 5 cm.; breadth, 3 cm.

Test remarkably thin, except on the tubes and the ridge connecting them, the latter being very thick.

Mantle moderately muscular over the branchial sac and on the tubes, membranous elsewhere. Tubes long, the branchial measuring 1·2 cm. and the atrial 8 cm. The former has a sharp bend dorsally and to the right above its middle, and at this point a

muscular process about 4 mm. long projects from the dorsal edge. Atrial tube more than half-way down the dorsal edge, and nearly at the posterior end of a crested ridge extending backwards from the branchial tube. The projecting points of the ridge are attached to the inner surface of the test. Right (branchial) side of the body long and narrow, left (visceral) very large, occupying all the ventral part of the body and even appearing on the right side below the branchial sac.

Branchial sac of moderate size, longish, pointed at the dorsal edge of the lower end; longitudinally plicated. The internal longitudinal bars placed on the ridges. Meshes square, with stout papillæ at their corners.

Dorsal lamina ribbed transversely, margin bluntly serrated.

Tentacles very numerous, crowded, long and slender, varying in thickness, but all of much the same length.

One specimen from Port Jackson, 6 fathoms.

Ascidia falcigera, n. sp.

External appearance.—Shape elliptical or nearly round, usually depressed. Area of attachment large, extending from the posterior end half-way up the left side; often expanded at the edge of the base into a thin spreading margin in which small stones are imbedded. Apertures on the upper (right) side, near the anterior end, not far apart. Branchial at or close to the ventral border, atrial near the centre—the latter is the more prominent though neither projects much; lobes very distinct, especially the atrial. Surface smooth and soft, slightly wrinkled. Colour from light-grey to pale horn tint, darker at the apertures. Length and breadth variable; as an average—length, 5 cm.; breadth, 4 cm.

Test thin, except at the base, where it is greatly thickened and has always gravel attached to or imbedded in it. Vessels large in the base, elsewhere few and of small size.

Mantle moderately muscular, especially on the tubes and down the centre of the right side. Tubes long, atrial much wider than branchial, which is bent towards the ventral edge in the middle of its length.

Branchial sac extending to the base of the mantle, not longitudin-

ally plicated. Transverse vessels all narrow. Papillæ only at the angles of the meshes, long, tapering, and curved like tusks. Broad membranes hang from the transverse vessels, and are stretched over the convex sides of the papillæ and attached to their apices. Three to five stigmata in a mesh.

Dorsal lamina very broad in its lower half, transversely ribbed, and minutely tuberculated at the edge.

Tentacles 35 to 40, long, crowded, their bases touching, of different lengths but not alternating.

Olfactory tubercle oval in outline.

Several specimens from Station 49 (South of Nova Scotia), 83 fathoms.

A few of the specimens are not so much depressed as the others, and have rather an oblong shape and terminal apertures.

This species shows considerable resemblance externally to *Ascidia obliqua*, but differs from it in the structure of the branchial sac, and especially in the form and arrangement of the papillæ.

Abyssascidia, n. gen.

Test cartilaginous, transparent. Branchial aperture about 12 lobed, atrial about 8 lobed. Attached by ventral surface.

Mantle thin. A few large distant muscle bands on the left side.

Branchial sac not longitudinally plicated.

Tentacles simple, filiform.

Viscera on the right side of the branchial sac, intestine small, stomach short and wide.

Reproductive organs forming a round mass situated on the right side of the intestinal loop.

Abyssascidia wyvillii, n. sp.

External appearance.—Shape oblong, rather pointed at the anterior end, rounded at the posterior end; attached to a small manganese nodule by the lower (ventral) surface in front of the middle; flattened so that, the branchial aperture being anterior, the atrial is on the upper surface three-quarters of the way to the posterior end, and rather to the right of the middle; in consequence of this, more of the left than of the right side enters into the formation of the upper

surface. Branchial aperture at the edge, slightly to the right of the anterior end, 12 to 14 lobed; atrial 8 or 9 lobed, both sessile. Surface smooth. Colour very light-grey, transparent. Length, 6 cm.; breadth, 4 cm.

Test thick, rather solid, transparent; no vessels. Consists of hyaline matrix and small fusiform cells, no bladder cells.

Mantle very thin, endostyle and viscera seen through distinctly. A few large distant muscular bands run round the right edge, and extend over the left side nearly as far as the endostyle. Atrial tube prominent and having fine muscle bands. Branchial also muscular, but not projecting.

Branchial sac large, fills the whole mantle cavity. Every second transverse vessel slightly larger than the intermediate ones; here and there the stigmata extend from the one large vessel to the other, cutting through the intermediate smaller one. The internal longitudinal bars widen at each intersection with a transverse vessel. Stigmata rather wide, three in each mesh. No papillæ at the corners of the meshes. Tusk-shaped ducts, to which horizontal membranes are attached, connect the transverse vessels with the swellings on the internal longitudinal bars.

Dorsal lamina reduced to a series of conical processes (languets).

Tentacles few, distant, small, and filiform. Two at each side of the anterior end of the endostyle, and a few others in the usual circle, but separated by nearly their own length from each other.

Viscera on the right side of the branchial sac, at the posterior end, relatively small.

Alimentary canal narrow. Œsophagus opens near the base of the branchial sac. Stomach short, wide, and barrel-shaped.

Reproductive organs forming a large rounded mass on the right side of the intestinal loop at the ventral end. The ovary occupies the centre, and the spermatic vesicles are arranged round the periphery. The oviduct and vas deferens emerge from the dorsal and posterior end of the mass, and course along the superior (anterior) margin of the intestine to their termination.

One specimen from Station 160 (South of Australia), 2600 fathoms.

This interesting form belongs undoubtedly to the Ascidiadæ

notwithstanding the large number of lobes round the apertures. It has affinities with *Ascidia* and *Corella.* The latter it resembles in the position of the viscera and in the shape and relative size of the intestine; the branchial sac on the other hand differs greatly from that of *Corella,* and exhibits the simpler structure found in *Ascidia;* while the membranes hanging from the transverse vessels and the languets replacing the dorsal lamina are exactly like the same parts in *Corella.*

Corella japonica, n. sp.

External appearance.—Shape longish ovate, the anterior end being narrower than the posterior. Attached by posterior end and half of left side; base sometimes prolonged into a few short tufts for attachment. Branchial aperture terminal or slightly on the right side of the extremity; atrial more than one-third of the way down and on the right side, nearer the dorsal edge than the middle. Both apertures sessile and inconspicuous. Surface slightly rough, especially at the anterior end. Colour grey. Length, 3 cm.; breadth, 1 cm.

Test thin.

Mantle very thin over most of the right side and lower half of left; while on the anterior end of the right side, the anterior (upper) half of the left side, and the tubes, muscular bands are extraordinarily developed, and attain a great thickness (up to 3 mm). Branchial and atrial tubes very muscular, the latter is the longer. Both apertures have ring-shaped ocelli of a light rust colour.

Branchial sac not folded. Internal longitudinal bars bear very long tapering papillæ. Stigmata curved, placed on the sides of conical infundibula set in square meshes. Secondary vessels coiled spirally, connected by a few radiating vessels. Broad horizontal membranes extend between the papillæ on the inner aspect of the sac.

Dorsal lamina in the form of languets.

Tentacles many, long, filiform.

Viscera on right side of branchial sac. Intestine large.

Two specimens from Yokohama, Japan, shallow water.

Several specimens from Kobé, Japan, 8 to 50 fathoms.

6. On some Applications of Rotatory Polarization. By Professor TAIT.

Since last meeting of the Society I have found that Broch (in Dove's *Repertorium*) employed the combination of prism, Nicols, and quartz plate, for the purpose of measuring the rotatory power of quartz for different wave-lengths. I do not find, however, that he suggests its use for the determination of wave-lengths according to one definite standard. Nor does he seem to have used great thicknesses of quartz, which is essential to accuracy in the application I have proposed. In the *Annales de Chimie*, 1846, there is a translation of a part of Broch's paper, with the remark that the process, which he called a new one, was due to Fizeau and Foucault. Their paper, however, refers to quartz cut *parallel* to the axis; but it introduces the question, very important so far as my object is concerned, of the interference of two polarized rays after one has been retarded more than the other by very many wave-lengths. It is quite possible that this consideration, which I had for the time forgotten, may be found fatal to my method when very great thicknesses of quartz are employed on the bright lines given by glowing gases, for the purpose of estimating the velocity of the individual particles. It will not affect the method as applied to the spectra of auroras, comets, &c.

Prof. Niven (*Phil. Mag.* 1878) speaks of rotation of plane of polarization as the most delicate test of change of wave-length. It is so in theory, but in practice it cannot be compared to a train of prisms.

I have within the last fortnight operated with pieces of quartz from 4 to 8 inches in length, and have found that the sharpness of extinction of the red line of lithium is greater than that of the green line of thallium. The breadth of the latter must of course come more into play. The range of uncertainty for the orange sodium line is very much greater still. This was to be expected from its being double. With thick plates of quartz it cannot be extinguished at all. In fact, in order to extinguish it, a plate would be required which would make the difference of rotation for the two constituents one or more semi-circumferences. The least thickness of quartz for such a purpose would be somewhere about 13 feet, and about 500 successive oscillations of the luminous particles would

have to be strictly periodic. The experiment would be well worth trying, but it would involve great difficulties as well as considerable expense; and it might fail altogether on another account, viz., the *breadth* of the individual sodium lines.

I find it advantageous to replace the second Nicol by a double-image prism, and to take the reading when the two images of the slit are equally bright.

BUSINESS.

The following candidates were balloted for, and declared duly elected Fellows of the Society:—J. M. Thomson, Esq., King's College, London, W.C.; Dr C. G. Knott, Natural Philosophy Laboratory, University, Edinburgh; Dr J. A. Russell, Woodville, Canaan Lane, Edinburgh; W. W. J. Nicol, Esq., 15 Blacket Place, Edinburgh; Charles Prentice, Esq., 8 St Bernard's Crescent, Edinburgh; L. L. Rowland, M.A., M.D., Williamette University, Salem, Oregon, U.S.; Robert Pullar, Esq., St Leonard's Bank, Perth; The Rev. Professor Flint, Johnstone Terrace, Craigmillar Park; De Burgh Birch, M.B., C.M., 19 Albany Street, Edinburgh; J. Berry Haycraft, M.B., B.Sc., Physiological Laboratory, Edinburgh.

Monday, 15th March 1880.

THE RIGHT REV. BISHOP COTTERILL, Vice-President, in the Chair.

The following Communications were read:—

1. The Topography of Jerusalem. By Lieut. Claude Reignier Conder, R.E.

The subject on which I have the honour of addressing you this evening is one far more complicated and difficult than that of the paper which I read to the Royal Society of Edinburgh some short time since. We have to deal, not with the surface of a country and the position of places of which the ancient names are still extant, but with a ruined city, buried to a depth of from 30 to 50 feet in rubbish on which modern buildings having been erected, and with

a topography in which there is scarcely a single important point which has not been controverted by one or more well-known writers.

The topography of Jerusalem has, moreover, formed the subject of works of every century from the age of Josephus to the present time, and can only be rightly understood after the study of about one hundred standard accounts of the city in all ages. Ruins, when discovered, must not rashly be assumed to belong to a period of great antiquity, since we know that even after the time of Herod the Great, magnificent buildings were erected by Hadrian, by Constantine, by Justinian, by the early Khalifs, and by the Latin kings, as well as by the Moslem rulers of the thirteenth, fourteenth, and fifteenth centuries.

Many relics which were at first thought to belong to the pre-Christian history of Jerusalem, have been shown, after careful examination by experienced architects, to be remains of the work of the later builders above cited. The subject of this paper must be confined, therefore, to an account of recent discoveries, and to an attempt at showing the bearing of such discoveries on the points of most general interest. The restoration of the ancient topography has, I take it, but small interest in itself except for a limited circle; but the questions which have given importance to the study are, I believe, very generally understood; and the controversies on Jerusalem topography appear to gather round two principal centres of interest—namely, the position of the site of Calvary and the Holy Sepulchre, and the position and the monuments of the Temple enclosure. It is proposed to show in what degree recent explorations have thrown light on these two central questions.

A new era in the history of Jerusalem research dates from the execution of the Ordnance Survey in 1864. This survey, undertaken partly at the expense of the Baroness Burdett Coutts (for the purpose of reporting on the water supply of the city), was executed by Captain (now Lieutenant-Colonel) C. W. Wilson, R.E., with a staff of non-commissioned officers from the Royal Engineers and was published by the Ordnance Survey Office at Southampton. The survey includes a map to the scale $\frac{1}{2500}$ of the city itself, with a smaller map of the envirous $\frac{1}{10000}$, and with a plan of the Temple enclosure $\frac{1}{500}$ as well as a number of special plans, of the Holy Sepulchre Church and other important buildings. The actual

levels of all the important points in the city were determined by instrumental measurement, and fixed definitely by connection with the line of levels which Colonel Wilson carried across from the Mediterranean to the Dead Sea.

This great work forms the basis on which all subsequent exploration rests, and supersedes all previous maps, such as those of Robinson, Vandevelde, Symonds, Catherwood, Barclay, &c., which, though invaluable at the time when they were constructed, are but imperfect attempts in comparison with the complete plan which is now available for students.

Colonel Wilson was not charged with any extensive commission for the purpose of excavations, and the few which he undertook did not lead to any important results. He was, however, the first explorer who gave a careful and minute account of the masonry of the great walls which surround the Haram or "Sanctuary," which is recognised as the site of the Jewish Temple enclosure.

The famous excavations conducted by Captain (now Lieutenant-Colonel) Warren, R.E., for the Palestine Exploration Fund, were commenced in 1867, and carried on until 1870. They resulted in discoveries of crucial importance, especially with regard to the extent and antiquity of the Haram rampart walls, and respecting the ancient contour of the site of the city and of the Temple.

The 1872 excavations were carried on at the expense of the German Government, in the very heart of the town, on the site of the old hospital of the Knights of St John. They also yielded very important results in the discovery of a great valley 100 feet deep, the existence of which had previously been denied by Canon Williams and other writers.

During the years 1872, 1873, 1874, and 1875, I spent several months of each year in Jerusalem, and was able to supplement the explorations in one or two particulars. I also collected, personally and by aid of residents, a large number of new observations as to the depth of the debris, which, when added to the measurements of Colonels Wilson and Warren, are sufficient to justify the tracing of contours over the entire site of the city, showing the ancient surface now hidden by rubbish, and thus defining the great natural features described by Josephus, which are almost obliterated by the gradual filling in of the original valleys.

A single instance of the value of such explorations may not be out of place here. Dr Robinson, the famous American traveller, discovered at the south-west angle of the Haram enclosure, the haunch stones of an ancient arch, and identified them as belonging to the bridge leading from the royal cloister to the upper city. Canon Williams challenged this assumption, and stated his opinion that Josephus should be understood to refer not to a bridge but to an embankment. The controversy extends over about twenty pages of print.

A single mine of Colonel Warren's set the question at rest, by the discovery of the great west pier of the ancient bridge, and of the voussoirs lying on the pavement 42 feet below the present surface, proving the existence of a magnificent viaduct 80 feet high with arches 42 feet span. Not content with this discovery, Colonel Warren broke through the pavement and sunk his shaft still 20 feet before reaching the rock, where, jammed in the channel of a rock-cut aqueduct, he discovered the voussoir of a yet older bridge, which had been overthrown before the pavement was constructed on an accumulation of 20 feet depth of rubbish. The earlier bridge is believed to be that mentioned as having been broken down at the time of Pompey's siege of Jerusalem; while the second viaduct, constructed by Herod the Great, was standing in the time of Christ, and was overthrown during the great siege of Titus.

Colonel Warren's explorations included a fairly complete examination of the southern, eastern, and western walls of the great enclosure of the *Haram* or "Sanctuary," and an examination of the passages, cisterns, and vaults in the interior.

The *Haram* is a quadrangle containing 35 acres, the interior surface roughly levelled, being partly rock, partly supported on great vaults, and partly filled in with earth, behind the great rampart walls. The four sides are of unequal length, the shortest wall being that on the south, 922 feet long. The south-east angle measures 92° 30,′ and the south-west is a right angle. The east wall is 1530 feet long, the west wall 1600, and the north 1042 feet.

In the north-west corner the rock has been cut down on the interior so as to leave a great block, 40 feet high, standing above the court, and now occupied by barracks. This rocky citadel measures 350 feet along the north wall of the Haram and 100 feet north and

south. The north face of the scarped block rises above an artificial trench, 50 feet deep and 140 feet wide, separating the Haram enclosure from the hill to the north.

The existence of a great viaduct at the south-west angle (identified with a bridge mentioned by Josephus) has already been mentioned. At the south-east angle, Colonel Warren made two further discoveries, both of which are of the highest value.

The great rampart wall here rises 160 feet from the rock foundation, of which height, however, only the upper half is at present visible above the surface. The ancient masonry is here found extending from the foundation almost to the top of the wall, and Colonel Warren found on the six lowest courses Phœnician letters, painted with a red pigment, which appear to have been intended to denote by a letter or numeral the course for which each stone was designed, beginning with the foundation course.

The second, and yet more important, discovery made by Colonel Warren at this point was that of a great rampart wall extending southwards from the Temple wall at the south-east corner. The two walls abut together with a straight joint, which extends from top to bottom without any bonding stones, indicating that they were probably built at different periods. The newly discovered wall was traced southwards for nearly 800 feet, and although it is nowhere visible above the surface, was found to be standing beneath the rubbish to a total height of about 70 feet. A great projecting tower was explored along the course of this wall, 400 feet south-west of the Haram corner, and there can be no reasonable ground for doubting that Colonel Warren was right in identifying this magnificent fortification with the city wall which Josephus mentions as protecting Ophel, while the great projecting tower answers in position to the "tower that lieth out" mentioned in the Book of Nehemiah also in connection with the Ophel wall.

The masonry of which the Sanctuary walls are composed may be divided into three principal varieties, belonging to distinct epochs of the history of the structure. First come the magnificent drafted stones, of gigantic proportions, forming the foundation of the wall and extending upwards generally higher than the present surface. It is undisputed that this masonry belongs to the period prior to the great destruction of Jerusalem by Titus. In the second place,

there is a distinct style above the drafted masonry, consisting of stones of large and remarkably square proportions without any draft. These stones are recognised by architects as belonging to the early Byzantine period. In the third place, there is a patchwork of smaller masonry, forming the upper part of the ramparts, and composed of Crusading, Saracenic, and Arab restorations, dating from the twelfth century to the present time.

It is with the two first classes of the masonry that we are at present concerned, and more especially with the drafted style.

The average height of a course of the drafted stones is about 3 feet 6 inches. The blocks vary considerably in length, the longest as yet known being 38 feet 9 inches and the second longest 23 feet 8 inches. The draft or sunk channel surrounding the four edges of the face of each stone is about 3 inches wide. The faces within this channel are generally dressed smooth, and project about half an inch in the best preserved specimens; but in the lower courses, where the stones were never exposed to view, the boss or raised face within the marginal draft is left undressed, and projects in some cases as much as 18 inches. The rampart walls are built with a batter or sloping face, and the draft allowed of great precision in the alignment of the stones even when the boss was left rough, the batter being obtained by setting back each course about 2 inches from the face of the course on which it stood, the measurement being taken from the face of the marginal draft.

Beyond the fact that this magnificent masonry belongs to the ancient Jewish Temple, no definite conclusion has as yet been generally accepted respecting its date. Colonel Warren has distinguished different classes of the masonry according to the finish of the stones, and refers these to different periods of construction. He attributes the stones at the foot of the east wall, close to the south-east angle, to the time of Solomon, while the unfinished masonry on the south-west he refers to Herod the Great. The well-known French explorer and architect, the Duc du Vogüé, is of opinion, on the other hand, that the whole of the drafted masonry at present existing is attributable to the time of Herod the Great, and although Colonel Warren's conclusions are generally well worthy of attention and singularly shrewd, there appear to me in this case to be many indications favouring the opposite view.

It is indisputable that the presence of Phœnician masons' marks at the foot of the great wall is proof of the antiquity of the structure; but there are good reasons for doubting whether Solomon's Temple enclosure can have extended so far south, while it is certain that similar characters would have been employed by native masons in the time of Herod the Great.

The masonry of the east wall of the Sanctuary towards the north is, moreover, of a somewhat different character and material from that of the south-east angle, and is attributed by Colonel Warren to a later epoch; yet, on this masonry also, red paint letters similar to those previously mentioned were found on the foundation stones of the wall.

There is, moreover, an indication of great value as to the date of the masonry yet to be noticed. The dressing of the stones is distinctive, and has not as yet been found elsewhere in Syria. An eight-toothed chisel was driven along the draft, and again used at right angles to its former direction, thus producing a regular criss-cross pattern on the surface of the stone. This dressing is found on each of the three ancient walls of the Haram, and on masonry of various degrees of finish, seeming to indicate a common period of execution for all the varieties of masonry, for it would be a bold assumption to suppose that the masons of Solomon used a chisel of exactly the same dimensions and number of teeth employed by Herod's masons, considering that a lapse of time equal to that which separates the reigns of Alfred and Victoria occurred between the two building epochs in question. The criss-cross dressing is found on the voussoirs of the Tyropœon Bridge, which, as before explained, has been found leading westwards from the Sanctuary wall, and which is plainly attributable to the Herodian period, to which it would therefore seem all the masonry similarly dressed is most naturally assigned.

The second type of masonry above the drafted stones has been attributed to Justinian by the same architectural authority above quoted, the Duc du Vogüé. Justinian is known to have erected splendid buildings on this site, and is the only builder between Herod and Omar who is historically recorded to have constructed anything on this side of Jerusalem. The architectural details accompanying the large smooth masonry in question are, moreover, identical in character with the style which is found throughout Syria in buildings of the fifth and sixth centuries, A.D.

Next in importance to the exploration of the Sanctuary ramparts must be ranked that of the subterranean passages and chambers within the enclosure, which were most carefully examined by Colonels Wilson and Warren, and the most important of which I have also frequently visited.

Two great passages lead from the two ancient gateways in the southern walls; two others lead from similar entrances on the west. The first two portals are each double, with an internal vestibule supported on pillars. The western gates are single, and the passages are half the width of the former two. The gateways in each case are ancient, with massive lintels above, having marginal drafts round the edges. The masonry of the passages, however, is in every case of more modern character, apparently belonging to the period of restoration under Justinian.

In addition to these vaults, there are no less than thirty large cisterns within the area, the aggregate capacity of which is calculated at about ten million gallons. Most of these great tanks are rock-cut, and some, which are closed at the ends with masonry and cemented inside, seem originally to have been passages like those above mentioned, but have been subsequently utilised as cisterns.

The measurements taken in the mouths and roofs of these cisterns have served to define generally the original rock surface of the ridge enclosed within the ramparts of the Sanctuary—a narrow spur running north and south with steep western slopes and more gradual eastern declivities. The rock at the north-west angle of the enclosure, standing 40 feet above the inner court, dominates the whole enclosure; but the ridge rises gradually to a point near the centre of the Sanctuary, where a rough rock surface is exposed beneath the beautiful "Dome of the Rock," at a level about 20 feet higher than that of the average surface of the enclosure. The broadest and flattest part of the ridge is found in the immediate neighbourhood of this rock, which forms the top of the hill included in the area. The level of the crest falls gradually southwards towards the tongue of land called Ophel, south of the Sanctuary; and the lowest point of rock within the area is at the south-east angle, where the foundation of the wall is 160 feet below the top of the *Sakhrah*, or sacred rock visible in the Dome of the Rock.

Such, briefly described, are the leading facts recovered with regard

to the Sanctuary at Jerusalem, so far as they have reference to the Jewish Temple. Briefly ennumerated they are as follows:—*First*, The existence of ramparts of gigantic masonry plainly attributable to the Jewish period and probably to the age of Herod the Great. *Secondly*, The recovery of a rock citadel with an outer fosse at the north-west angle; of an ancient viaduct of the Herodian period at the south west-angle; and of the great Ophel wall abutting on the Sanctuary at the south east-angle. *Thirdly*, The existence of two southern gateways of the Jewish period and of two western portals, in addition to the entrance over the bridge and other passages in the interior of the area. *Fourthly*, The determination of the rock-levels throughout the area, and the fact that the Sacred Rock occupies the culminating point of the broadest part of the Temple ridge.

These facts have all more or less important bearings on the great question of the restoration of the Temple enclosure, which is one of the most important subjects of controversy in the topography of Jerusalem. Authorities are at present divided into two parties, the largest of which recognises in the present Sanctuary enclosure, as a whole, the area of the Jewish Temple as restored by Herod; while the smaller party, following the teaching of Mr James Fergusson, supposes that Herod's Temple occupied only a square of 600 feet side in the south-west portion of the area.

The reason for this last assumption is the statement given by Josephus, that Herod's Temple enclosure measured a furlong on either side—approximately 600 feet; and until it had been ascertained that the eastern wall, and the northern and eastern parts of the west and south walls of the Sanctuary (wherever examined) were of antiquity equal to that of the ramparts towards the south-west part of the area, such a theory had many points in its favour. The recovery of the great Ophel wall has, however, proved to be the most important of the many valuable discoveries made by Colonel Warren, because Josephus has clearly stated that the Ophel wall joined the *east* wall of Herod's Temple, just as the rampart now found joins the east wall of the present Haram. The existence of a rock citadel and fosse on the north-west answers also exactly to the account which Josephus gives of the citadel of Antonia, which dominated the Temple courts; and three angles of the ancient enclosure are thus identified with corresponding angles of the

modern area, through the recovery of remains of a citadel, a bridge, and a city wall, described by Josephus.

The recovery of the north-east angle is, however, not as yet complete, although Colonel Warren has been able to throw light on this question also. The north wall of the present enclosure is generally acknowledged to be later than the other three, and consists of very inferior masonry. The east wall has, moreover, been proved to run northwards without any break beneath the surface, beyond the present north-east angle of the enclosure; but, as already mentioned, this part of the east wall is of inferior material and finish though marked as ancient by the existence of the ancient red paint letters. It seems probable that this wall formed part of the rampart erected by Agrippa about 44 A.D. on the north side of Jerusalem, which wall joined the east rampart of the Temple enclosure. There are indications, which cannot now be given at length, tending to show that the old north-east angle of Herod's enclosure was situated about where the Golden Gate (an edifice of the Byzantine period) now stands, and that an area of about 2½ acres in the north-east portion of the present enclosure was included within the boundaries at a period later than that of Herod's Temple.

The above remarks apply exclusively to the Second Temple as restored by Herod the Great. Although it is certain that the Holy House itself and the altar occupied the same spot in the time of Herod on which they were first reared by Solomon, the extent and position of the surrounding courts as they existed in Solomon's time are as yet entirely unknown, no certain remains of that period having been recovered, and no definite accounts of their measurements being extant. We know that great alterations were effected by Herod; that he increased the area (Josephus says in one passage that he doubled it) and that he took away ancient foundations and laid others. Considering the lapse of twenty-nine centuries, and the alterations deliberately effected at the late period, it seems improbable that we should succeed in restoring the Temple of Solomon, though there seems no reason why the main features of that of Herod should not be recovered with certainty, in the process of explorations, which are still both necessary and possible.

A second objection to the proposed restriction of the Herodian Temple within a square of 600 feet side in the south-west portion

of the existing area, arises when the levels of the rock surface are carefully studied.

The Temple enclosure consisted of three principal courts, rising in successive steps towards the great fane which stood, as Josephus states, on the top of the hill. In order to fit such a building to the ground, within the present area, it is necessary to start with the assumption that the culminating point is to be found in the Holy Rock—the present top of the Sanctuary Hill. If we place the Holy House over this rock, the levels of the various courts agree exactly with the ascertained levels of the rock as at present remaining; but if we were to place the Holy House further south-west it would have stood, not on the top, but half way down the steep western slope of the mountain. The lowest court would be found to occupy the highest part of the hill; and foundations varying from 50 to 90 feet in depth would become necessary on the supposition that the great edifice was built up from the rock.

The restoration of Herod's Temple on the supposition that the central fane stood above the sacred rock, called *es Sakhrah*, in the middle of the present Dome of the Rock, has occupied my attention for more than seven years past; and the indications which confirm this restoration are perhaps sufficiently interesting to claim a detailed enumeration.

In the first place, The rock in question has been regarded by Jews, Christians, and Moslems, for at least fifteen centuries as the site of the Holy of Holies. We learn from the Talmud that a stone or rock called "Foundation" formed the floor of the most holy place, and that it was regarded as the foundation of the whole earth. From early Christian writers we gather that this sacred rock was venerated by the Jews in the fourth century, and the description then given tallies with the present Sakhrah. The traditions of the Crusaders and the Arabs reproduce those of the Jews in regarding the present Sakhrah as the foundation stone of the world, and thus serve to connect the present sacred rock with that on which the Temple stood.

Secondly, The Temple faced eastwards, and its door, according to the Talmud, was directly opposite the summit of Olivet. A line drawn due east through the Sakhrah rock will be found, if produced, to strike the top of Olivet, which would not be the case were the Holy House placed further south.

Thirdly, The levels of the various courts can easily be deduced from the Talmudic account of the Temple; and as the floor of the Temple is fixed at the level of the Holy Rock, the levels of the surrounding areas may be compared with those of the present enclosure. In every case the result is satisfactory The Court of the Priests ought, on the present theory, to have been 2432 feet above the Mediterranean, and an observation of the rock at exactly that level has been obtained within its area, the level being only a foot beneath the present flat surface of the court round the dome of the rock. The Court of the Women should have a level 2429, and it is almost certain that the rock is nowhere above (nor very much below) that level within its precincts. The Court of the Gentiles should be 2411 above the sea, which is the average level of the present surface outside the platform or court round the Dome of the Rock. Several other exact results might be given, but the preceding will be sufficient to show how well the ancient Temple may be fitted to the ground surrounding the Holy Rock.

Fourthly, There were no cisterns within the limits of the inner courts of the Temple; and none of the great cisterns which exist in so many other parts of the Sanctuary enclosure come within the limits of the courts according to the present restoration.

Fifthly, A subterranean gallery leading to a subterranean bath-house ran northwards from the great gate Moked on the north-west side of the Priests' Court; and such a gallery with an adjoining vaulted chamber is found in exactly the required position according to the present restoration.*

It should be noted that the arrangement of the courts which has been followed is that given in the Talmud, which agrees fully with the more general account of Josephus; and that the cubit is assumed to have been about 16 inches in length,—a determination which, as I had occasion to notice in a former paper, is based from a comparison of Talmudic accounts with existing monuments, especially the Galilean synagogues and the masonry of the rampart walls of the Sanctuary above described.

If the above views should be found tenable, discoveries of the

* The identification of these vaults with the passages mentioned in the Talmud, is due to Colonel Warren, whose plan of the Temple is, however, somewhat different from that proposed in this paper.

highest interest may perhaps still await us. It is known that the pavement of the platform round the Dome of the Rock is partly supported on vaults as yet unexplored, and that there are rocky scarps still to be examined in other parts of the same area. It seems quite possible that the rocky foundations of Herod's Temple may still lie hidden beneath the modern pavement; and the progress of exploration during the last twenty years has been so steady and rapid, that it is perhaps not unreasonable to hope that the secrets of this hidden portion of the sacred area may yet be unfolded, and the position of the Holy House and the Court of the Priests fixed by the actual recovery of the foundations at present covered by a modern pavement.

While residing at Jerusalem I entered every chamber and cistern which could be reached in the immediate vicinity of the Holy Rock. I was fortunate in the discovery of a rock scarp previously unnoticed, but there is no doubt that substructures remain still to be explored especially towards the east of the present platform.

Such seems to be the general result of the successive explorations of the High Sanctuary at Jerusalem, so far as Jewish antiquities are concerned. It is now proposed to consider the results of exploration in the city itself, in special connection with the question of the true sites of Calvary and the Holy Sepulchre.

The first requisite for a satisfactory restoration of the ancient topography of the city is a clear understanding of the natural site on which it was built. The hills and valleys have been rendered almost indistinguishable through the accumulation of rubbish; and modern explorers have therefore agreed that the first and most important object to be kept in view is the determination of the level of the rock-surface in all parts of the city.

The Ordnance Survey of Colonel Wilson not only gave the means of easily determining such levels by reference to fixed bench marks above the surface, but also laid the foundation of the inquiry by marking the rock wherever it occurred on the surface. The number of observations added by Colonel Warren and by others, including my own measurements, has given a total of 265 observations within an area of about 250 acres occupied by the ancient city; and we are thus able to run contours and draw sections which approxi-

mately determine the original surface within very narrow limits of error, and which render the relative positions and elevations of the principal features as certain and definite as is necessary for antiquarian purposes.

The most important discovery resulting from these researches is that of the great valley which occurs in the very heart of the city, having its head not far from the Jaffa Gate. The rock appears in the Church of the Holy Sepulchre in two places at a level 10 feet above the floor, but just south of the church there are vaults 18 feet deep.

The rock is also known in many places along the top of the hill called Zion by the modern Christians, but between these two eminences it is never visible on the surface, which is somewhat depressed.

Dr Robinson pointed out that there was apparently a valley separating the southern hill from that on which the Church of the Holy Sepulchre stands. Yet this was so little capable of proof before the levels of the Ordnance Survey had been taken, that Canon Williams did not hestitate to affirm that no such valley existed.

In 1872 the excavations on the site of the Hospital of St John resulted in the discovery of magnificent vaults, 50 feet deep and 200 feet long, which formed vast reservoirs for the supply of the Hospital. These were cleared out to the bottom, and the rock was found at a level 60 feet below the top of the Holy Sepulchre hill and traced all along the vaults.

In 1876 another vault was found further east, measuring 120 feet north and south. The rock floor was found to fall rapidly southwards, and the slope of the north bank of the great valley was thus defined over a considerable section. In 1870 Colonel Warren had made observations which define the position of the south bank, and the number of observations in and near the valley now number about thirty in all. The general result is, that its course is traced eastwards to the Haram, where it joins a narrower valley running north and south from the Damascus Gate. The newly recovered valley is 60 feet deep and 600 feet wide north and south.

The existence or non-existence of this important natural feature used to form one of the favourite subjects of Jerusalem research. Those who, following Dr Robinson, continued to believe in its

existence were stigmatised as *Tyropœonists,* from the theory that this was the course taken by the valley which Josephus calls the Tyropœon. But not even the party which now proves to be right was prepared for the great width and depth of the valley.

Having thus recovered the most important of the lost physical features of the site, we are better able than of old to understand the description given by Josephus, which is almost too well known to need repetition. Josephus speaks of three hills and two valleys within Jerusalem. First, of the great square hill, on which the upper city stood in his time; which he identifies with the citadel of David's time, called in the Bible the "stronghold of Zion." The lower city occupied the slope of Akra—a hill less elevated than the former, having a gibbous or bulging shape, and divided by the Tyropœon valley from the first mentioned hill of the upper city. Another valley separated Akra from a third and lower hill, which was called Bezetha, and which was situated north of the Temple hill, with an artificial trench cutting it off from the Antonia citadel.

All these features are recovered. The elevations all prove to be relatively those described by Josephus, the shapes of the hills, the dividing valleys, even the artificial rock-cut trench, are found correctly to describe existing features.

The large square hill, which Josephus incidentally identifies with Sion, is that now called by the same name. The flat plateau on the summit is 2530 feet above the Mediterranean. North of the great dividing valley a spur, 40 feet lower than Sion, is found bulging out eastwards. It is divided from the hill north of the Temple by a second valley which joins the first; and the positions of Akra, Bezetha, the Upper City, and the Tyropœon, are in the opinion of Colonel Warren, and I may be perhaps permitted to add in my own opinion also, now defined in the relative positions indicated some forty years since by Dr Robinson.

The course of the ancient walls which surrounded Jerusalem is carefully described by Josephus. On the south-east and west the city was defended by deep valleys, but on the north there were three consecutive lines of defence at the time of the great siege. The remains of these ancient fortifications seem, however, for the most part to have disappeared, and the only traces which we may con-

fidently expect to recover are the rocky scarps which formed the base of the walls in various parts.

The modern walls of Jerusalem are partly composed of ancient drafted masonry, which is, however, not *in situ*, and there seems good reason to suppose that the materials of which the present fortifications are composed were taken from the ancient walls. The ramparts have been destroyed and rebuilt seven times since the siege of Titus, and the disappearance of all traces of the ancient third wall is most easily explained, on the supposition that its stones have been removed, since there is no great accumulation of rubbish to hide any remains which might exist north of the city.

Even within the last half century many relics of the ancient city have been lost for ever. Dr Robinson speaks of the remains of towers north-west of the city, which have now entirely disappeared beneath modern buildings. There can be little doubt that these represented the course of the third wall, and the careful measurements and angular observations of Dr Robinson are thus of the highest value. In 1864 other remains were noted, during the execution of the Ordnance Survey, of ancient masonry, which has since been broken up by the peasantry. The accumulation of rubbish in Jerusalem has in fact been of the greatest service to antiquarians, and where no rubbish exists the ancient buildings have been entirely destroyed.

The first wall of Jerusalem defended the upper city. At the north-west angle was a fortress with three famous towers, Hippicus, Phasaelus, and Mariamne, which stood on solid bases and protected Herod's palace.

There can be little doubt that the present citadel south of the Jaffa Gate marks the site of this fortress. The great tower now called David's Tower has been proved to stand on a solid base. Its dimensions are almost exactly those given by Josephus for Phasaelus, and it resembles that tower also in having an outer platform with a battlemented wall. The north-west tower of the citadel may mark the site of Hippicus, but the present structure is rather larger than the tower which Josephus describes as the corner tower of the first wall on the north-west.

The modern citadel is surrounded by a fosse, and east of this is the market place standing on arches, the rock being 30 feet below the surface of the street.

The vaults are entered by a door from the fosse, but this is now built up. The examination of these vaults would be an undertaking of the greatest interest as tending to throw light on the course of the first wall.

From the corner tower Hippicus the wall ran east to the Temple. The rock levels now obtained show the existence of a precipice or scarp running eastwards from the vicinity of the citadel; and on this line, also, the foundations of two ancient towers have been discovered, which seem to have formed part of the north face of the wall.

The south-west angle of the ancient city is now recovered in a satisfactory manner. A rocky scarp, which had long been observable, was thoroughly explored in 1874 by an English engineer, Mr. Henry Maudslay. During my stay in Jerusalem I made a careful survey of the remains discovered. The rock was found to have been worked to a vertical face to a height of 50 feet for a distance of 150 yards. At either end of this scarp was a projecting rock buttress, the base of a tower 40 feet square. A flight of rock-cut steps led up to each tower from the fort of the scarp, and numerous cisterns were excavated in the rock on the top of the tower bases.

It is interesting to note that this arrangement agrees exactly with the description which Josephus gives of the towers along the wall of the ancient city.

The rock scarp is found to continue beyond the towers both northwards and eastwards. The towers stand in the precincts of the Protestant cemetery and bishop's school, and Mr Maudslay was unable to obtain leave to continue his researches beyond the limits of this property. There can be little doubt that one of the most useful and interesting researches remaining to be undertaken, consists in the following out of this discovery, and the further tracing of the ancient wall foundation.

The manner in which the first wall joins the Temple enclosure at the south-east angle has already been described. It is not, however, as yet known exactly where the wall crossed the Tyropœon valley. The account given in the Book of Nehemiah and that of Josephus are both too indefinite to be clearly understood without the aid of explorations along the line. A careful tracing eastwards

of the scarp already recovered might, however, probably result in the settlement of this question.

The second wall is very briefly described by Josephus. It started from a certain gate in the first wall called Gennath, which, appears to have been at no great distance from the tower Hippicus and it ran thence *in a curve* to Antonia, enclosing the lower city. The gate Gennath has not yet been found, and this also is one of the great future objects of research. No certain relics of this wall indeed have been as yet recovered, for the remains of drafted masonry which Canon Williams found east of the Church of the Holy Sepulchre have been very carefully examined, and prove to have belonged to a building which the Duc du Vogüé has shown to have been part of the ancient Basilica of Constantine, built in 335 A.D. over the supposed site of the Holy Sepulchre. The northern scarp of the rock-cut fosse which separated the tower Antonia from the northern hill of Bezetha has been traced westwards for some distance. It seems very probably to have formed the counterscarp of a ditch outside the second wall. Colonel Warren also found remains of a rocky scarp facing northwards within (or south of) the line of the above mentioned counterscarp, but this investigation is as yet incomplete, and a shaft is much needed within the precincts of an open plot of ground immediately west of the scarped rock of Antonia.

The materials of which the third wall was composed seem, as already remarked, to have been removed by the builders of the later walls of the city. The general line of this wall, which was built about 44 A.D. by Herod Agrippa, has been laid down by Colonel Warren in a manner which appears to me to be satisfactory.

Starting from Hippicus this wall ran out northwards to a certain large octagonal tower called Psephinus, which stood on ground so high as to command a view of the mountains of Arabia. The remains of towers were discovered by Dr Robinson along this part of the course of the wall, but are now hidden or destroyed. From the high ground at the point now occupied by the Russian Cathedral, the mountains of Arabia east of Petra were distinctly visible when covered with snow in the winter of 1873–4.

From the tower Psephinus the wall ran east and then south-east and passed over certain caverns called the "Caverns of the Kings."

Most modern writers identified these caves with the great quarries which extend under Jerusalem immediately east of the Damascus Gate. These quarries were those whence the Temple masonry was hewn, and a curious rude carving of a figure resembling the Assyrian winged bulls has been found in them and is now in England.

From this point the line of the third wall seems to have coincided with that of the present north wall of Jerusalem, standing on a rocky scarp with a rock-cut fosse outside. The modern eastern wall, as far south as the Sanctuary enclosure, also appears to be on the same line occupied by the east face of the third wall, which thus joined on to the comparatively roughly finished wall which, as above mentioned, runs northwards beyond the present north-east angle of the Sanctuary.

The main interest of tracing these ramparts lies in the connection of their course with the question of the genuineness of the site now shown as representing the Holy Sepulchre and the Hill of Calvary. It is admitted that if the remains of the second wall can be shown to have included these sites within the boundary of the then existing city, the description of the position of Calvary outside Jerusalem—as plainly set forth in the New Testament—would not be fulfilled. The question, however, of the course of the second wall is still unsettled.

Without wishing to enter into this old and fierce controversy at the very end of my paper, I would point out three indications which arise out of the recent discoveries.

First, The hill on which the present Church of the Holy Sepulchre stands is now identified, apparently beyond dispute, with that of Akra or the lower city, which was encompassed by the second wall.

Secondly, The deep valley separating this hill from that on the south runs up almost to the Jaffa Gate. The second wall started from some unknown point on the north side of the first wall and ran in a curve to Antonia. It seems impossible to suppose that it can have *crossed through* the great valley, and if it was built on the high ground at the head of that valley, and ran thence in a curve to Antonia, it must apparently have included the Holy Sepulchre Church.

Thirdly, The site of the church is beyond dispute within the compass of the third wall, which was built to protect suburbs which had extended beyond the second wall. It is true that the second

wall only existed in the time of Christ; but the third wall was built only ten years after the Crucifixion. It seems difficult to believe that the suburb, in the short period, had extended itself over 120 acres, so as nearly to double the area of Jerusalem. It seems more probable that at the time of the Crucifixion the site now shown as Calvary was already, if not within the walls, at least far within the limits of the existing town.

To state briefly the objection, raised first in the eighth century and repeated by various writers in almost every succeeding age, the site now shown as representing Calvary is so nearly in the middle of Jerusalem, that it seems impossible, on any reasonable reconstruction of the ancient city, to suppose that at the time of the Crucifixion it was outside the border of the inhabited town.

One of the strong arguments in favour of the genuineness of the site has always been found in the existence close to the Holy Sepulchre of an indisputably ancient Jewish tomb. I propose to inquire, therefore, in conclusion :—First, What is this tomb, and how is its presence inside Jerusalem to be understood? Secondly, If the site of Calvary was not in reality where it is shown, where it is likely to have been?

As regards the tomb, it is a chamber cut in rock, with nine *kokim* or graves, of which three are placed at a lower level, sunk in the floor of the chamber. The fact that the graves are *kokim*, that is longitudinal tunnels, running in from the sides of the chamber, so that the body lay with its feet towards the chamber and its head away from it, and that they are not *loculi*, or graves placed sideways on the walls of the chamber, proves not only that the tomb is Jewish, but that it belongs to an early Jewish period previous to the time of Christ.

We are informed by the Talmudic writers that all the tombs were outside Jerusalem, except the tombs of the nine kings of Judah, and another tomb of the prophetess Huldah. Josephus tells us that the graves of these kings were hidden, so that even those standing inside the monument could not see them. The site of the Tombs of the Kings has long been anxiously sought, for the present traditional site is recognised as having been invented in the fifteenth century. The ancient tomb above described answers all the requirements of the tombs of the kings.

1*st*, It is an ancient Jewish tomb.

2*d*, It is within Jerusalem.

3*d*, It contains graves for nine kings, which was the number buried, including David and Solomon.

4*th*, It is the only known Jewish tomb inside the city, ancient or modern.

This view, which is, I believe, original, has already been cordially accepted by many students of the question. It thus furnishes an argument against instead of in favour of the present site of the Holy Sepulchre.

As regards the probable site of Calvary, I have also in conclusion to mention a new indication.

It is agreed that Calvary and the Holy Sepulchre were close together and outside the town, and it is generally supposed that Calvary was the place of public execution.

The tomb of Joseph of Arimathea, where Christ was laid, was in a garden, as are still the tombs of wealthy personages, but it was not the less likely to be near the great cemetery of the town. After careful investigation, and the recovery of inscriptions, frescoes, sarcophagi, and other remains, it has been pretty clearly shown that the ancient Jewish cemetery of Jerusalem was on the north of the town. The southern cemetery is Christian, and there are very few ancient Jewish tombs on the east. On the north, among the gardens which still extend over the flat ground, as described by Josephus, there are many ancient tombs, including that of Simon the Just. It is in this direction apparently that the Holy Sepulchre should be sought, though it is probably now beyond the power of modern research to identify out of so many sepulchres that of Joseph of Arimathea.

Calvary was, we may perhaps assume, the place of Jewish public execution. The recovery of the place of execution is therefore a matter of the highest interest. In the Talmud the place is described under the name "House of Stoning," as being just outside the city. It appears to have been a precipice some 12 feet high, over which the culprit was thrown before the first stone was cast at him. The site of this place is still pointed out by the Jews. It is a precipice with a swelling mound or hill above, and a cavern in the cliff, which is known to Christians as Jeremiah's Grotto. The

site in question is just outside the Damascus Gate, and can be easily shown to have been outside even the third wall of ancient Jerusalem.

There seems no doubt that this is the ancient place of execution, and I leave it to your consideration, whether it may not reasonably be supposed to be the site of Calvary, "the place of a skull."

In conclusion, I would call your attention to the immutability of the topography of the Holy City. The Rock of Foundation still stands in a Temple, and within a sacred enclosure. The citadel of Antonia is still a citadel. The Upper and Lower Markets are still markets. The Royal Towers still form the western fortress of the town. The venerated tomb of the kings of Judah is surrounded by a famous cathedral; but the site of Calvary—the place of execution—is only graced by a cemetery with an adjoining slaughter-house; and the private sepulchre of the rich man is still indistinguishable among the number of rich men's tombs in the gardens beyond the city walls.

If there are any here interested in the subject of further exploration at Jerusalem, I would point out that much yet remains to be done. The second wall has to be found. The great discoveries on Ophel and Sion require to be followed up, the secrets of the enclosure of the High Sanctuary are not exhausted, and mines are much needed in the ground immediately west of Antonia.

It is my earnest hope some day to be enabled to take up the task of excavation from which such great results were obtained by the energy and skill of Colonel Warren, and I trust that should such an opportunity arise the funds may not be wanting; for the difficulties arising from the dangers of the work were counted as but small, in comparison with the pecuniary obstacles which had to be met during the whole of the period during which Colonel Warren was so bravely persevering in the recovery of monuments which have an undying interest for the whole Christian world.

2. The Geology of the Faröe Islands. By James Geikie, LL.D., F.R.SS. L. & E.

The author visited the Faröe Islands last summer in company with Mr Amund Helland of Christiania. They made various

traverses across the largest and most important islands, and touched here and there at several of the smaller ones. They have constructed a geological map of the group, upon which is shown the outcrop of the coal-seams of Suderöe, the direction of numerous dykes of basalt, the position of great intrusive sheets of the same rock; and the trend of the glaciation is indicated by arrows. The introductory part of this paper gives some account of the geological observations made by previous writers—Jorgen Landt in 1800, Mackenzie and Allan in 1815, Trevelyan a year or two later, Forchhammer in 1824, Robert Chambers in 1854, and Johnstrup in 1873. The general physical features of the islands are next described, the extent of land being roughly estimated at about 600 square miles. Nearly all the islands have an elongated form, and are drawn out in a N.N.W. and S.S.E. direction. This is likewise the direction of the more or less narrow sounds or open fiords that separate the islands in the northern part of the archipelago, as also of the wider belts of water in the south. All the islands have a mountainous character, and everywhere exhibit, in the most marked manner, the well-known terraced outline which is so common a feature of trappean masses, the highest elevation they attain is 2852 feet, but many of the hills approach to within 200 or 300 feet of that dominating point. The mean elevation of the northern group of islands is estimated to exceed 800 feet, and is probably not less than 900 feet. The coasts are usually precipitous, many of the cliffs exceeding 1000 feet, and in some places even 2000 feet in height. The valleys are described as ascending from the sea in a series of great steps or terraces—each terrace being cirque-shaped and framed in by a wall of rock, the upper surface of which stretches back to form the next cirque-like terrace, and so on in succession until the series abruptly terminates at the base, it may be, of some precipitous mountain. Occasionally the *col* between two valleys is so level that it is difficult to detect the actual water-parting. In this case the two valleys combine to form a kind of deep hollow passing right across the island from sea to sea. Lakes are very numerous, but of small size, and the streams are also abundant but of inconsiderable importance.

The author then goes on to describe the geological structure of the islands, which is extremely simple. The rocks consist principally of bedded basalts with intervening layers of tuff, and in Mygenæs

and Suderöe of clay, shale, and coal. The prevalent dip of the beds in the northern islands is south-easterly, but in Mygenæs it is easterly, and in Suderöe, or the southern island, north-easterly. Nowhere does the strata incline towards the west. The angles of dip are generally very low—in the northern islands not averaging more than 2° or 3°, while in Suderöe they are a degree or so higher.

The oldest part of the series is represented in Suderöe and Mygenæs. The basalt rocks in these islands consist principally of bedded anamesites, composed of plagioclase, augite, magnetite, and olivine. Their behaviour in the field and the general aspect they assume are described in considerable detail. They are all more or less amygdaloidal. Very often the various sheets of old lava are separated by partings and layers of a red fine-grained palagonitic tuff. Near the top of the anamesite series occurs an irregular belt or band of shales and clay with two seams of coal. [The position of this belt was shown upon a coloured diagram, representing a section across the island of Suderöe.] A thin and local seam of coal and shale is found much lower down in the series. [This also was shown upon the diagram.] The distance between this local coal and the workable coal-beds above is about 1100 feet. The author next gives a detailed description of the various outcrops of the coal, and traces its extension over Suderöe. [The more characteristic appearances presented by the coal were shown in another diagram.] The two workable coal-seams vary in thickness from a few inches up to two or three feet respectively. They are mined to a small extent, and in a very primitive manner.

The author next gives a particular account of the basalt rocks above the coal. It is these rocks which compose the major portion of the northern islands; the only one of the north islands which shows any trace of the coals and the lower igneous series being Mygenæs. The basalt rocks above the coals are for the most part more coarsely crystalline than those of the older series. They are dolerites rather than anamesites, but their composition is the same. They are also as a rule more coarsely amygdaloidal—the cavities often reaching a great size—two feet and even more in diameter. The minerals they contain are chiefly chabasite, stilbite, apophyllite, analcime, quartz, calcedony, calcspar, and green-earth, and it is not uncommon to find two, three, or even four different zeolites in one

and the same cavity. After describing in detail the various appearances assumed by the dolerites, and the features exhibited by the associated palagonitic tuffs, the author gives some account of the intrusive basalts, which are of two kinds—one occurring in sheets intruded along the line of bedding, the other in mere thin dykes and veins.

He estimates the thickness of the anamesites to be not less than 4000 feet, and that of the dolerites as between 9000 and 10,000 feet; thus giving a total thickness for the bedded volcanic rocks of not less than 13,000 or 14,000 feet. As the dip of the strata is extremely regular, and there are no large dislocations to complicate matters, this estimate may be relied upon as approximately correct.

He then enters into a lengthened discussion as to the origin of the strata, and combats the prevailing belief that the igneous rocks are relics of submarine eruptions. The conclusion come to is that the bedded basalt rocks of the Faröe Islands represent the heavy basic, and more liquid lavas which flowed from a cone or cones placed at some considerable distance, probably to the west of the present islands. The palagonitic tuffs, which sometimes contain small stones and grit, and are often laminated, represent partly fine volcanic dust (which the winds could carry considerable distances), partly volcanic mud, and to some extent they may also have been derived from the subaerial disintegration of the exposed lavas. They pass here and there into regular shales and tuffaceous clays, especially upon the horizon of the coal-seams. The coals are composed of land plants of Miocene age, and many plant remains occur in the associated clunch or clay and shale. None of these appear to have grown *in situ*—there are no roots penetrating an ancient soil. The coals are made up of the debris of plants carried down by freshets into shallow pools and marshy meres. Not a trace of marine organisms was observed in any strata throughout the islands. The coals and clays indicate a pause in the volcanic activity, during which the Miocene flora invaded the igneous area; but whether from the direction of America or Europe it is impossible to say. The local seam of coal at Dalbofos which occurs low down in the series, proves that there were more than one such pauses, and its fragmentary condition leads to the suspicion that the Miocene flora may have again and again invaded the

region, during long pauses between the eruptions;—all trace of these invasions having subsequently been destroyed by newer flows of lava.

The glacial phenomena are described in great detail under the following heads:—1. Glaciation; 2. Till or boulder-clay; 3. Erratics and Morainic debris; and 4. Rock-basins.

1. *Glaciation.*—Every island visited showed conspicuous marks of glacial abrasion; and notwithstanding that the rocks have suffered much since the glacial period from the action of the weather, striæ are yet well preserved in many places. They are very plentiful in and round Thorshavn, where they point E. 35° to 45° S. An examination of Stromöe and Osteröe, which were traversed in several places, and the smaller islands lying to the north-east, proved that the whole northern group had been buried under a thick sheet of ice, forming one compact *mer de glace* which flowed out in all directions from the dominant points of the islands. The glaciation was traced up to a height of 1600 feet, and as the water in some of the fiords is 100 fathoms deep, we must add this to the other measurement to get the maximum thickness of the ice (2200 feet), which flowed outwards from Faröe. The extreme northern ends of the islands were sought everywhere for indications of any invasion by ice from the direction of Greenland, but no trace of this was found; on the contrary, the rocks were there highly rubbed, polished, and striated in a direction from south to north. In Suderöe the glaciation goes up to 1400 feet; above that elevation the rocks are harsh, rugged, and serrated, just as they are above the limits of glaciation in the northern islands. The southern island showed that it too had been nearly smothered in ice, which moved off in all directions; and not only so, but it was evident from the direction and position of the striæ in the north of Suderöe that its *mer de glace* was coalescent with that of the northern islands. During the glacial period the Faröe Islands were thus united by one and the same ice-sheet, which had no connection with either the *mer de glace* of Greenland or that of Northern Europe. It was entirely a local ice-sheet flowing outwards from the main elevations of the islands, and breaking off all round, no doubt, in icebergs.

2. The Till is exactly comparable to the boulder-clay of Scot-

land, Scandinavia, and Switzerland. It is a more or less local accumulation of angular, subangular, and striated and smoothed stones and boulders, set in a matrix of hard gritty earth and clay. Its composition, its position with regard to the configuration of the ground,—sheltering as it does in the lee of rocks, whose polished faces look in the opposite direction,—and the mode of its distribution mark it out as the *moraine du fond* of the old *mer de glace.* Every stone it contained belonged to the islands, not a single fragment of any rock foreign to the Faröes occurring in it.

3. Erratics and Morainic debris are scattered about everywhere, and mark the retreat and gradual disappearance of the ice-sheet. Many of these erratics attain a large size, some measuring upwards of 20 feet across. Not one of them is foreign to Faröe. Few well-marked terminal moraines in the valleys were observed, partly owing to the fact that great quantities of debris have fallen from the cliffs, and tended to obscure the glacial debris heaps, and partly because they have suffered much from the action of torrents and freshets. Here and there, however, mounds of morainic origin are conspicuous enough.

4. The Rock-basins are next described, and their origin assigned to the grinding action of the glaciers. They are numerous upon the land, and seem to be as common a feature of the fiords of Faröe as they are of the Scottish and Norwegian sea-lochs.

The latter part of the paper discusses the origin of the valleys and fiords, which is ascribed partly to subaerial erosion and partly to glacial excavation. The origin of the main water-parting of the islands also comes in for discussion, and considerable space is devoted to the consideration of such topics as present atmospheric and marine erosion in the islands. The paper concludes with some account of the peat with its remains of small trees, and a description of a number of typical rock specimens.

3. By special permission of the Society, there was read a Meteorological Note by Mr Alexander Wallace. Communicated by Professor Piazzi Smyth.

On Monday, March 1, 1880, a strong breeze from the S.W., marked at 1 o'clock P.M. in meteorological observation, as equal to 15 miles

per hour, prevailed, after which the force of the wind rapidly increased, shifting to W.N.W. until about 2 o'clock when it became a perfect hurricane, accompanied by a storm of snow and sleet. The velocity of the wind must have been betwixt 40 and 50 miles per hour, and partly from heavy black clouds covering the sky, and partly from the dense sheet of snow which was drifting along, everything became obscured.

From the point where I stood (at the door of my house, the Old Observatory, Calton Hill) there appeared two currents of snow-drift, one on each side, which meeting each other about 20 feet in front of me, after a severe struggle coalesced and shot upwards with excessive velocity slantingly, and towards E.S.E. As that upshot was going or gone, but when the storm was still at its maximum, a loud crash was heard, and a vivid flash of light simultaneously seen, much as one may suppose would be the effect of the bursting of a bomb-shell within a few feet of you.

Almost immediately after this the storm began to abate, and the remainder of the day was comparatively calm. Altogether the storm did not last above ten minutes, and during that time the barometer fell $\frac{2}{10}$ths of an inch, but rose again immediately afterwards to its former height.

4. On the Colouring of Maps. By Professor Tait.

(*Abstract.*)

Some years ago, while I was still working at knots, Professor Cayley told me of De Morgan's statement that four colours had been found by experience to be sufficient for the purpose of completely distinguishing from one another the various districts on a map.

I had previously shown that if an even number of boundaries meet at each point on a diagram, two colours (as on a chess-board) will suffice for the purpose. But in a map, boundaries usually meet in threes.

I replied to Professor Cayley that I thought the proof might be made to depend upon the obvious proposition that not more than four points in a plane can be joined two and two by non-intersecting lines. Here points were made to stand for districts. When two such points are joined by a line they must have different colour-titles. I

did not at the time pursue the subject, as I found that it was more complex than it appeared at first.

Mr Kempe's paper in *Nature* (February 26, 1880) has recalled my attention to the subject, and some simple modes of treating the question have occurred to me. The germs of them are in what I have said above, and they show one easily how to proceed to colour any map. A sketch only of one of them is now given.

Begin by making as above stated a companion diagram, putting points for districts, and lines joining them for common boundaries. Then by introducing (in any way) as many new joining lines as possible (but so that no two intersect) the diagram is divided into three-sided compartments.

Next, make all of these compartments four-sided by taking a number of new points, each on a joining line. The whole set of points can now be lettered A and B alternately, because two colours suffice for a map whose boundaries meet in fours. But let the intruded points be lettered *a* and *b*, instead of A and B respectively.

Now perform the same operation in a second way, differing everywhere from the first, and call the newly intruded points α and β instead of A and B.

Rules are laid down for carrying out these operations; but they require too many illustrative cuts to be given here.

Then any one triangular compartment will appear in two essentially different forms: for instance, with its intruded points it may read (in the two cases, taking the corners in the same order) B, *a*, B, A and B, A, β, A. Now superpose the two figures, lettering included, and attend to the order of the two letters at the same point. We have, from the instance above, the compound reading (attending now to the corners only) BB, BA, AA, of which the separate terms are necessarily different. Hence every point in the figure is lettered differently from all that are joined to it, and only four designations can occur, viz.: AA, AB, BA, BB. This proves the proposition, and gives one mode of colouring the original map. For the *erasure* of joining lines (such as were originally introduced to divide the whole into three-sided compartments) does not necessitate any change of lettering.

This mode of treating the question shows incidentally that in a map where only three boundaries meet at each point, the boundaries

may be coloured with *three* colours, so that no two of the same colour are conterminous.

This particular process essentially introduces four different colours, and therefore does not necessarily give the simplest way of colouring a map. Another method, quite different from this, but involving virtually the same principles, is next given. Then come two other processes, different in form from that of Mr Kempe, but based like it ultimately on the fact that only $3(n-2)$ non-intersecting lines can be drawn (except for $n=2$) joining n points in a plane.